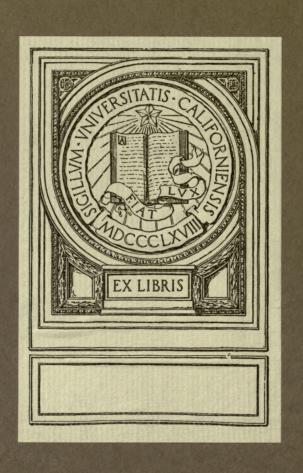
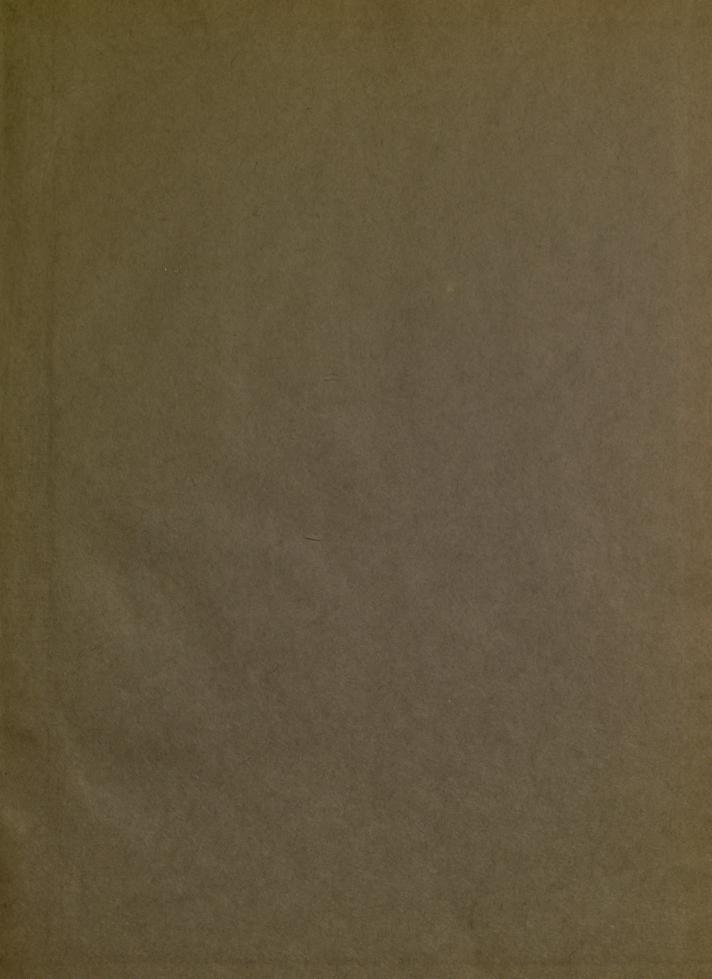
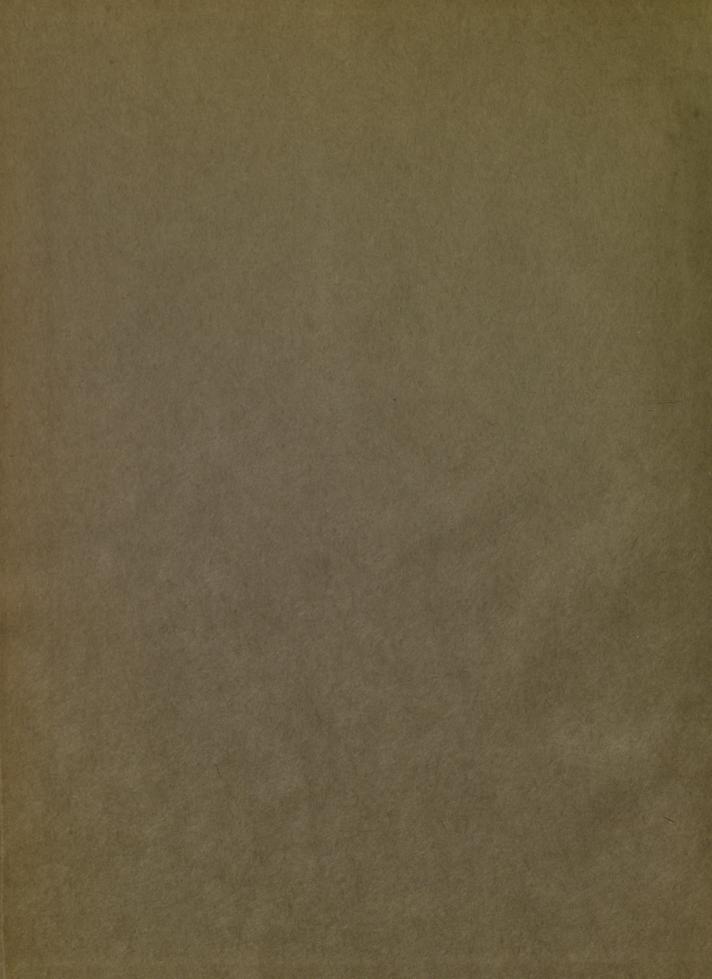
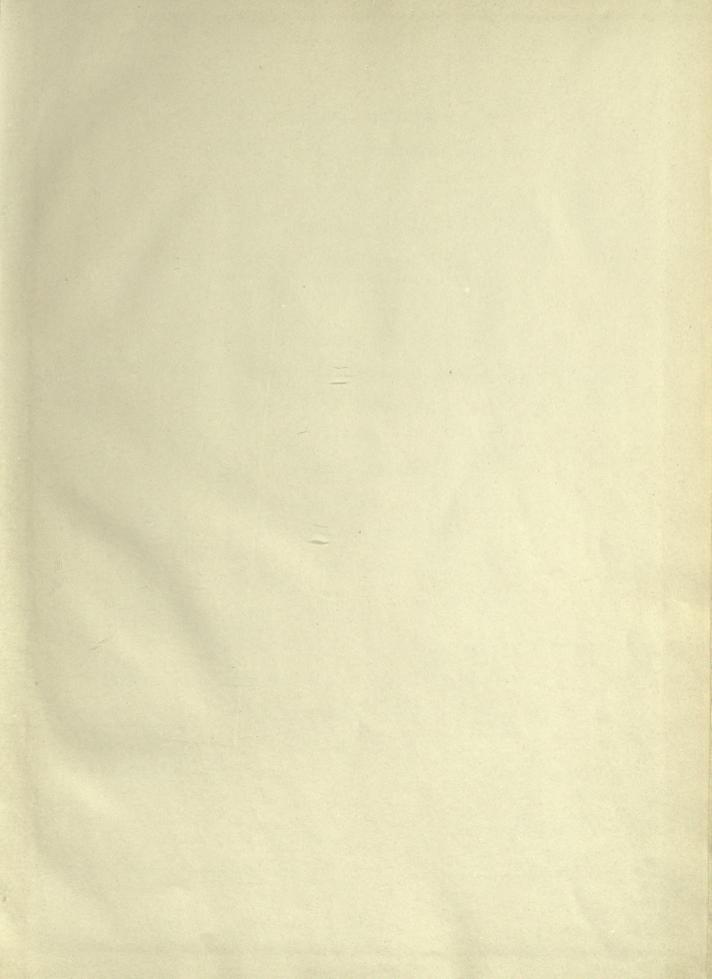
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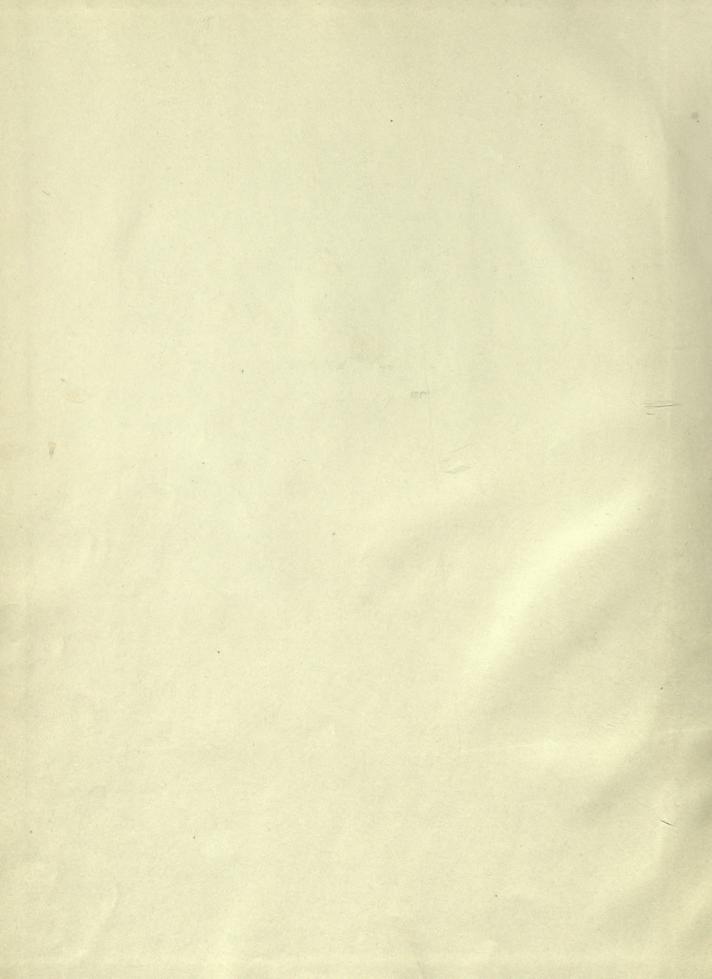
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THE ANALYTICAL DEPERMINATION OF ELEXTRIC RAILWAY SPEED - TIME RELATIONS

By

Lloyd Nash Robinson

B.E. (Union College) 1911 M.S. (University of California) 1917

THESIS

Submitted in partial satisfaction of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in the

GRADUATE DIVISION

of the

UNIVERSITY OF CALIFORNIA

June, 1919

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INTRODUCTION

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The design of a modern electric railway necessarily involves an accurate predetermination of the performance of the completed system when the construction period shall have passed and the traffic begins to move. Consequently, the speed of trains predominates among the factors to be treated. The speed of a train, like that of any moving body, varies as a function of time in accordance with the fundamental laws of mechanics. From these principles, it is possible and practicable to predetermine the speed of a given electrically propelled train at any instant in the course of a run over a known or assumed track. For high speed roads with frequent stops, such as subway systems, the converse problem of determining the time required to attain a particular speed may be of greater importance and of correspondingly more frequent occurrence.

The predeterminations of speed-time relations necessarily depend on the type of motive equipment and therefore must be based upon the characteristics of the motors in the case of electrically propelled trains. The methods, in current use for carrying out these determinations are step-by-step, graphical processes. The most used

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The present purpose is to derive formulae, by means of which the speed-time relations may be determined directly by arithmetic substitutions and operations without recourse to graphs other than the characteristic motor curves. Since the continuous current series motor is used almost universally in American electric railway practice, only the formulae applicable to this type of motor are presented although the method of derivation should indicate how analogous formulae, suited to other types of motors, may be derived.

As a foundation for the subsequent derivations, a segregation of the factors involved in electric propulsion of trains is essential.

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FACFORS IN MLECTRIC TRAIN PROPULSION

A - ENERGY, POWER AND TRACTIVE EFFORE

The movement of a car or train along a track necessitates an expenditure of energy. In the general case, the mechanical energy, supplied to a train, simultaneously serves three well defined purposes, and consequently must be treated as the sum of three distinct components. These components are: first, that dissipated due to friction; second, that stored or liberated as potential energy due to change in the elevation of the train; and third, that stored or liberated as kinetic energy due to change in the speed of the train.

By the application of what is known as dynamic braking, that is, operating the motors as generators while descending grades or while making stops, mechanical energy may be extracted from a moving train, converted to electric energy and returned to the electric distribution system. Physically, the extraction of energy is the reverse of the process of supplying energy. Thus, in calculations, energy extracted must be treated as negative energy supplied. Hence, the energy supplied in a chosen period of time will be positive or negative according to operating conditions.

The dissipation of energy due to friction continues as long as a train is in motion. The dissipated energy can not be recovered as useful mechanical energy in subsequent operations since it passes off as heat or possibly in other, less easily discernible forms. Its name well defines it as lost. The dissipated energy in turn may conveniently

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be resolved into three components. These components are: first, that due to friction in traveling along level tangent track, technically referred to as due to train resistance; second, that due to an increment of friction introduced by track curvature, usually spoken of as due to curve resistance; and third, that due to the application of friction brakes, known as due to braking effort.

Potential energy is stored while a train is ascending a grade.

During a subsequent descent, potential energy is liberated and either extracted, dissipated or transformed to kinetic energy. Kinetic energy is stored while the speed of a train is increasing. As the speed later decreases, kinetic energy is liberated and either extracted, dissipated or transformed to potential energy, depending on whether the decrease in speed is due to dynamic braking, friction or to ascent of a grade.

The potential energy of a body is the same when it is at the same elevation. Also the kinetic energy of a body at stendstill is zero. Therefore all the energy, that is supplied to a train during its run between two stations which are at the same elevation, is ultimately dissipated before the train stops unless some of it is recovered by means of dynamic braking.

Although the foregoing discrimination of the forms, in which energy is expended, is an essential step in the analysis of train propulsion, the term, speed-time relation, accurately implies instantaneous values of speed and time. Consequently the determination of these relations is immediately concerned with instantaneous rates of energy expenditure, consumption and absorption. That is to say, it depends upon the input and distribution of mechanical power, for power is the instantaneous rate of energy expenditure, conversion, consumption or

be reactived into three components, these components are; first, that due to inicited in traveling along level tempent track, technically referred to as two to train resistance; eaces, that due to as increased of friction introduced by track ourvature, werelly spoken of as one to curve resistance; and third, that due to the replication of friction broken, brown as due to braing affect.

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absorption. The mechanical energy input has been divided into three main components. Similarly, the mechanical power input is the sum of the corresponding instantaneous rates of energy dissipation and storage.

The measure of mechanical power input to a moving body is the algebraic product of the applied force and the instantaneous speed at which the point of application of the force is moving. In an electrically propelled train, the mechanical power input is transferred from the motors through gears to the axles at the surface of the latter. Hence the measure of the mechanical power input to an axle is the product of the tangential force at the surface of the axle and the peripheral speed of that surface. However, in railway calculations, it is convenient to use the speed of the train as a basis of reference so far as possible.

The peripheral speed of a point on the tire of a car wheel is the same as the speed of the car provided the wheel does not slip. The peripheral speed of a point on the tire of a wheel is to the speed of a point on the surface of its axle in the ratio of the diameters of the wheel and axle. Therefore the ratio of the train speed to the peripheral speed of the cyclindrical surface of the axle is equal to the ratio of the diameters of wheel and axle.

By the principle of moments, a tangential force at the surface of an axle may be replaced by an equivalent force applied at the surface of a concentric cyclinder of greater radius. The tangential force, which must be applied at a radius equal to the wheel radius, in order to produce the same moment as the actual tangential force applied at the surface

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sucception. The cooleanical creaty input has been divided into three east of main components. Similarly, the medicalcel power imput is the east of the corresponding instantaneous rates of energy dissipation and charges.

The electrons product of the applied force and the instantements aged to a which the product of the applied force and the instantements aged at which the point of application of the force is taying. In an electrically proposited train, the medical paper input is transferred from the motors through seams to the exist at the carriede of the latter. Hence the measure of the pack and the production from the train and the production of the train and the product of the train areas and the partaments of the train areas of the axis and the partamental appeal of the appeal of the train as a latter of trains age that a convenient to use the appeal of the train as a heats of reference so far as possible.

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of the axle, must be to the latter in the inverse ratio of the diameters of the wheel and axle. This equivalent force, hypothetically acting on the axle at a radius equal to the semi-director of a driving wheel, is called the tractive effort.

The above relations can be expressed concisely in algebraic form: thus

= Motor tractive effort X train speed .

per hour, and that of tractive effort is pounds, avoirdupois. The total tractive effort applied to a train is the sum of the tractive efforts applied at the several driving axles. However, for purposes of comparison and for simplicity in computations, the tractive effort is usually referred to the weight of the train. That is, the tractive effort is spoken of as so many pounds per ton of gross train weight. The phrase, pounds per ton, unfortunately contains a latent ambiguity because the number of pounds in a ton is presumably constant, but there should be no confusion when the expression is confined to railroad parlance.

Because of the proportionality of mechanical power and tractive effort, the applied tractive effort is obviously divisible into several components corresponding to the rates of energy dissipation and storage.

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B - COMPONENTS OF TRACTIVE

In general, the tractive effort applied to a train may be resolved into the following components: resistance duy to the side

- 1 Corresponding to the rate of energy dissipation,
 - (a) Train resistance.

 - (b) Curve resistance, (c) Braking effort of friction brakes;
- 2 Corresponding to the rate of storage of potential energy, (a) - Grado resistance; 0 = 0.5 D to 0 = 1.5 D
 - 5 Corresponding to the rate of storage of kinetic energy. (a) - Accelerating effort.

the triols and wheels and upon the degree of superalevation of the

Train resistance to the degree of the survey that he the manes of

When a train is moving on level tangent track, there is energy dissipated in rail and journal friction, air resistance, etc. The sum of the forces, corresponding to the rates of emergy dissipation due to these causes, is called the train resistance. The magnitude of the train resistance at any instant depends upon the train weight, speed, cross-sectional area, and the number of cars. The relations of these factors have been determined by extensive experiments with different classes of equipment in operations under wide ranges of conditions. From the experimental data, empirical formulae for train resistance have been deduced. One of these, which is quite commonly used, is that developed by Mr. A. H. Armstrong, namely:

$$R = \frac{50}{\sqrt{T}} + 0.03 \text{ V} + \frac{0.002 \text{ X}}{T} \left\{ 1 + \frac{H-1}{10} \right\} v^2,$$

in which

R = Train resistance in pounds per ton of gross train weight,

T = Gross weight of train in tons (2000 lbs.),

V = Speed of train in miles per hour.

X = Projected cross-sectional area of train in square feet.

N = Number of cars in train.

$$\frac{50}{\sqrt{m}} \geq 3.5$$

I - COMPONING OF TRACETYS SPECIED

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Curve Resistance

when a train is proceeding along a horizontal curve in the track, there is introduced an additional resistance due to the side pressure of the wheel flanges on the rails, the unequal distribution of weight of the train on the two rails, etc. This additional resistance is called curve resistance.

The curve resiscance varies from C = 0.5 D to C = 1.5 D pounds per ton of gross train weight, depending upon the condition of the track and wheels and upon the degree of superelevation of the outside rail. D is the degree of the curve; that is, the number of degrees of central angle subtended by a one hundred-foot chord of the circular arc described by the center line of the track. It is clear that, for average conditions, C may be taken as numerically equal to the degree of the curve.

Braking Effort of Friction Brakes

When friction brakes are applied to the wheeels of a moving train, energy is dissipated in accordance with the laws of sliding friction of metal on metal under pressure. However, since the pressure of the brake shoes is subject to the immediate control of the operator, the magnitude of the braking effort does not bear any fixed relation to the train speed. In applications of the brakes on long downgrades, the braking effort is kept practically constant; and, when service stops are being made, the braking period is of such short duration that no serious error in computations is introduced by treating the braking effort as constant during this period. Considerations of the cafety of the equipment and of the comfort of passengers dictate the

off of the course of a proposition of the course of the the course of the the train of the train of the train of the course of t

The curve realization varies from U = 0.6 p to 0 = 1.5 p pounds per ten of each train votate, depending upon the candition of the track and when the degree of superelevation of the outelfarail. It is the degree of the curve, that is, he makes of the degrees of central angle subtended by a one handred-feet chord of the chord of the tentral and the the central line of the track. It is cheer that, for average contibious, I may be totan se materically open to the degree of the curve.

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allowable braking effort. In subsequent formulae, the braking effort of friction brakes is represented by B and, like the other components of tractive effort, it is measured in pounds per ton of gross train weight.

Grade Resistance

In mounting a grade, a train absorbs energy which is returnable when the train later descends to a lower elevation. The component of tractive effort, corresponding to the rate of storage, or liberation, of potential energy, is

where 2000 is the number of pounds in a ton, and \$\vec{g}\$ is the vertical angle measured upward from the horizontal to the grade line of the track.

Since the per cent grade, 100 tan p, can readily be obtained from the surveyors' data, it is convenient to express the grade resistance as

For light grades, such as are met in steam railroad practice, there is no appreciable error introduced by assuming that

and using the approximation,

G = G = 2000 (Per cent grade) = 20 X Per cent grade.

But much heavier grades are common in urban and suburban electric systems and serious error may be introduced in calculations by applying the approximation.

allowable braking effort. In embraquent formulas, the training effort of frietien braken is represented by D and, like the other components of bracking effort in pounds per ton of prosest train weight.

Crade Resistance

In mounting a grade, a train absorbe energy which is returnable when the train later decounts to a lower elevation. The compound of tractive effort, corresponding to the rate of atorage, or liberation, of potential energy, is

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for high grains, and as are not in steem railroad practice, there is no supresimited that

and value the approximation,

but much heavier grades are common in when and educates against against the supremiseation.

while train resistance, curve resistance and the application of friction brakes all tend to retard the motion of a train, it is evident that grade resistance will tend to retard or to accelerate depending on whether the train is proceeding uphill or down. This is taken into account in the formulae by assigning to the value of G the positive or negative algebraic sign as diotated by the above considerations. That is to say, the value of G is positive or negative according as that of sin Ø is positive or negative.

If the magnitude of the propelling force applied to a body is greater than that, under the action of which the existing speed of the body will be maintained, the speed of the body will increase; and, conversely, the speed will decrease if the propelling force is insufficient to maintain the extant speed of the body. Consequently, if the tractive effort, applied to the driving wheels of a train, is more than sufficient to overcome the train resistance, curve resistance, braking effort and grade resistance, the balance will operate to accelerate the speed of the train. This component of the tractive effort may be termed the accelerating effort.

The accelerating effort, necessary to produce an acceleration of A miles per hour per second, is usually taken as 100 A pounds per ton of gross train weight. The value of the accelerating effort is positive or negative depending on whether that of A is positive or negative; that is, according as the speed of the train is increasing or decreasing.

while brain remissance, ourse contained and the application of ristless brains all tend to retard the motion of n train, it is evident that the process and the conclusion dependents from the training and whether the training to the related to the pales is below into account in the formulae by assigning to the value of G the paritive or magnifus algebraic of an elective of a the positive or negative as any, the value of G is positive or negative according as that is to ear, the value of G is positive or negative according as

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Summary of Components

The algebraic sum of the above five components is the tractive effort applied to the driving axles through the functioning of the motors and gears. That is to say,

in which

F = Tractive effort applied at driving axles,

A = Acceleration of train speed in miles per hour per second,

B = Braking effort of friction brakes,

C = Curve resistance,

G = Grade resistance.

R = Train resistance.

The unit of measure of all, except the acceleration A, is points (avoir dupois) per ton (2000 lbs.) of gross train weight.

C - TRAIN ACCELERATION

Transposing F and A in the above tractive effort equation,

$$A = 0.01 (F - B - C - G - R)$$
.

This relation expresses the fact that the acceleration is determined by the values of the applied tractive effort, braking offort, and curve, grade and train resistance.

The acceleration is the instantaneous time rate of change of the train speed; that is.

$$A = \frac{dV}{dt}$$

Hence

$$\frac{dV}{dt} = 0.01 \ (F - B - C - G - R) .$$

Substituting for R its equivalent, given on page 7 above,

$$\frac{dV}{dt} = 0.01 \left\{ F - B - C - G - \frac{50}{\sqrt{T}} - 0.03 V - \frac{0.002 X}{T} \left\{ 1 + \frac{N-1}{10} \right\} V^2 \right\}$$
 (1)

Brancher of Comments

The algebraic sum of the above five components is the tracking after through the functioning of the motors and groups.

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in which F = Tractive effort applied at driving axles.

A = Acceleration of Srein speed in miles per hour per second,

B - Braiding effort of friction brains.

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E - Train reministrate.

the unit of measure of all, except the encelaration A, is pounds (avoir dupois) per ten (2000 les.) of gross truit weight.

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of train speed. Its solutions render formulae for the direct prodetermination of the speed—time relations for known or assumed service conditions. As has been previously stated, B, C and G are independent of the train speed V. However, the applied tractive effort
F may vary as a function of the train speed as will be seen from a
consideration of the characteristic curves of railway motors and the
usual methods of operating them.

performed in been which are usually made at the factory. Next is moved the characteristic current of a conditional entreet railony meters. For mornal voltage applied to the motor barmistics, their
corresponding the relations between the motor surgest and and of
the fallowing factory:

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of train speed. The solutions remain formulae for the Sirat prodependential of the speed with reflections for instal or secured dorviou conditions. In her been previously stated, B, C and G are independent of the train speed V. However, the applied tractive effort I may vary as a function of the tests speed as will be seen from a consideration of the entractoristic ourses of reflecy motors and the

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RAILWAY MOTOR CHARACTERISTICS

The series motor derives its name from the fact that its field circuit is connected in series with the armature circuit as is shown in Fig. 1. It is clear that the motor current, armature current and field current are identical.

The performance of a motor under operating conditions is most conveniently expressed in what are known as characteristic curves. These curves for railway motors are determined from their performance in tests which are usually made at the factory. Fig. 2 shows the characteristic curves of a continuous current series railway motor. For normal voltage applied to the motor terminals, these curves display the relations between the motor current and each of the following factors:

- 1. Speed of car or train in miles per hour.
- 2. Tractive effort of motor in pondis,
- 3. Efficiency of motor and its gears in per cent,
- 4. Power output of motor with its gears in kilowatts.

It is clear that the relations, expressed by the curves, are all affected more or less by the diameter of the driving wheels and by the reduction ratio of the gears between motor and exle. Also the relation of speed to current is largely dependent upon the motor terminal voltage. For these reasons, the gear ratio, wheel diameter and terminal voltage, for which the curves apply, are stated on the curve sheet.

The speed-current curve shows that the current decreases as the speed increases. This is due to the direct proportion connecting

BALLERY MOTOR CHARACTERISTICS

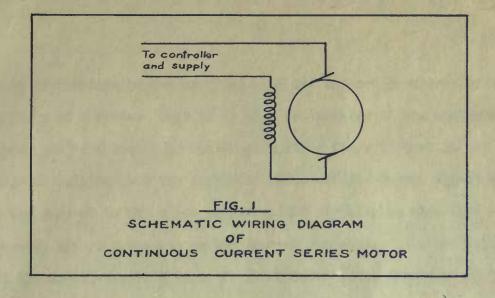
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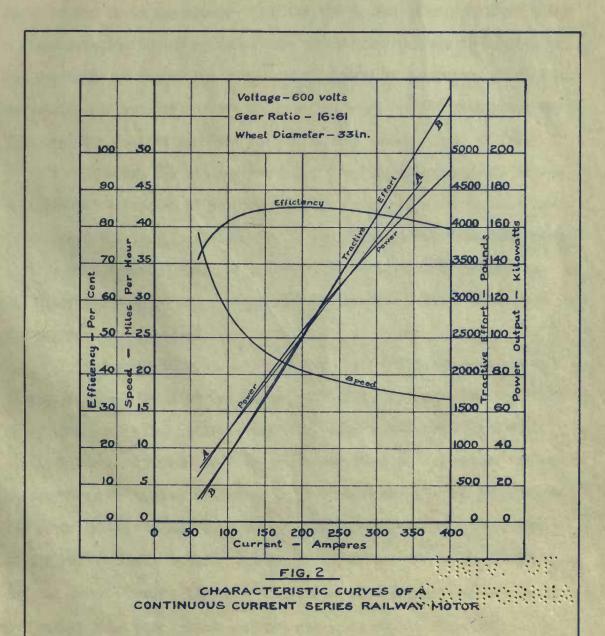
The performance of a motor under operating conditions is most conveniently expressed in what are known as characteristic ourves. These curves for railway motors are determined from their performance in teats which are namally under at the factory. Fig. 2 characteristic curves of a continuous curvest earlies railway motor. For normal voltage applied to the motor terminals, these curves display the relations interes the motor current and each of the following factors:

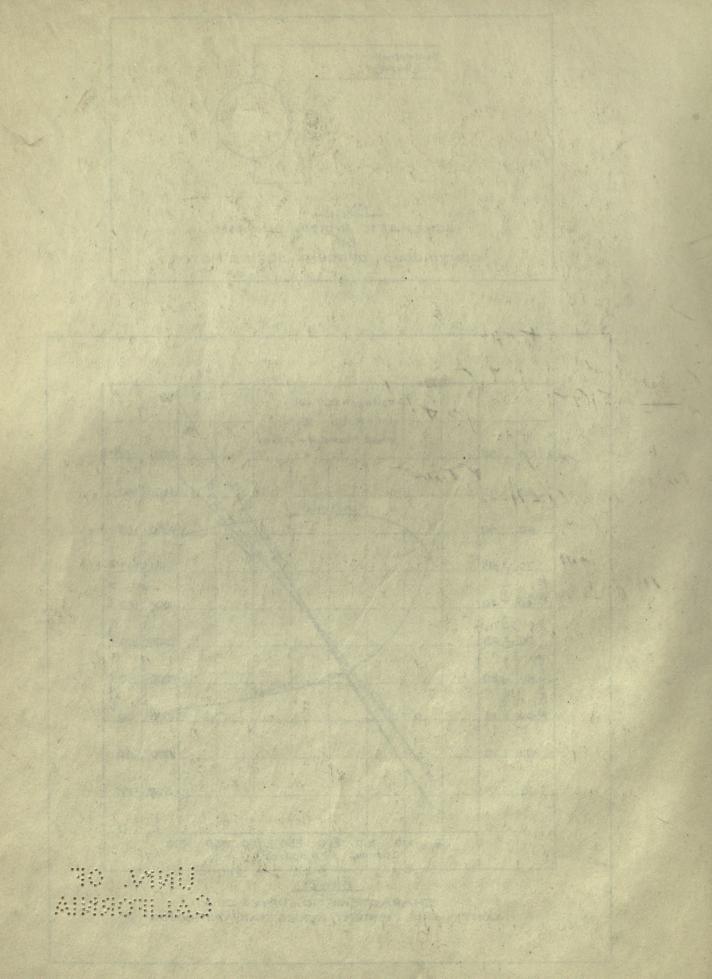
- 1. Speed of our or train in miles yes hour,
 - animison of motor to traits evident .
- 3. Efficiency of motor tend the general in yet demt.
- 4. Fower orstynt of motor with its genra in Milowates.

It is also that the relations, expressed by the curves, are all affected more or less by the diameter of the Arriving whose and by the object of the respect of the genes between motor and anilo. Also the relation of speed to comment is integrily depondent upon the motor because yellongs. For those reasons, the gene redte, wheel diameter and terminal voltage, for which the durves seely, are stated as the curve alact.

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the speed of rotation of the armature with the induced counter-electromotive force of a motor. That is to say, an increase of the armature
speed produces an increase of the electromotive force induced in the
armature and this opposes the impressed electromotive force, thus reducing the current in the motor circuit. The curves show also that
the tractive effort decreases as the current decreases. The net result
is that the tractive effort input to the driving axles decreases as the
speed of the train increases. In other words, the tractive effort bears
a fixed relation to the speed as long as constant voltage is applied to
the terminals of the motor. This relation must be determined so that an
expression for tractive effort in terms of speed can be substituted for
F in the differential equation (1) before the latter can be solved.

The constitut of the line A4 in Fig. E is

Although the speed-current and tractive effort-current curves indicate the existence of perfectly definite, continuous relations between speed and tractive effort, it is impossible to express these relations in a formula rationally derived from fundamental principles. The alternative is to use an approximation that is sufficiently exact for engineering purposes.

The tractive effort-current and power output-current curves
in Fig. 2 — and these curves are typical in this respect although they
apply to a particular motor — may be closely approximated by the
straight lines AA and BB over the operating range of the motor. It has
already been shown that the power input to the train, which is equal to
the power outputs of all the motors combined, is the algebraic product
of the tractive effort and train speed. By making use of these relations
and the above straight line approximations, an algebraic expression of
the relation between tractive effort and speed may be obtained.

The speed of reterion of the areabore with the induced compensation motive force of a motor. That is to say, as increase of the equation speed in the speed produces as increase of the electromotive force, thus remaind and this opposes the inpressed electromotive force, thus remaining the current in the motor circuit. The curves show also that the tractive effect decreases as the curves show also that inductive effect input to the diving also decreases. The first the tractive effect bound is the diving a constant vertex boars a time tractive effect increases. In other words, the tractive effect boars a fixed relation to the appeal of the tractive effect in the relation must be determined so that any each of the tractive effect in turns of apold and is applicated for apprearion for tractive effect in turns of apold and is applicated for a the discontinal equation (1) before the latter can be colved.

Although the speed-courses and tractive offert-courses continues the course of perfectly definite, continues relations bear indicate the expense thing and tractive effort, it is impossible to express thing re-lationally derived from fundamental principles. The elternative is to use an approximation test is antitionally exact for engineering purposes.

The treative effect courses and power outpost in this respect although they apply to a particular notes -- may be closely approximated by the etraight lines it and ill ever the operation of the motor. If has already been shows that the power imput to the train, which is equal to the power outputs to the train, which is equal to the power outputs of all the motors contined, is the algebraic product of the treative affect and train epoch. By making use of those relations and the above affect and training of the short and tractive expressions of the power attention and tractive expressions, an algebraic expressions of the relation between tractive expressions and the above attention of the approximation of the obtained.

The equation of the line AA in Fig. 2 is

in which

P' = Power output per motor in kilowatts,

I = Current per motor in amperes,

h' and h' are constants determined by the co-ordinates of any two points on the line AA.

To reduce the power to kilowatts per ton of gross train weight, substitute

p' = ha + hal

$$\frac{T}{N}P = P^*$$

in which

P = mechanical power input to driving axles in kilowatts
per ton,

T = Gross weight of train in tons,

M = Number of motors in the train.

Then
$$\frac{T}{M}P = h_1^2 + h_2^2I \tag{4}$$

or
$$P = \frac{M}{T} (h_1^2 + h_2^2 I)$$
 (5)

Since F is expressed in pounds per ton and V in miles per hour, and since it is desired to express the power in termsof F and V, the power, P kilowatts per ton, must be reduced to mile pounds per hour per ton. One kilowatt is equivalent to

$$503 = \frac{550 \times 3600}{0.746 \times 5280}$$
 mile pounds per hour. (6)

Thus the power input to the driving axles is

Hence
$$F V = \frac{503 \text{ M}}{T} (h_1^4 + h_2^4 I)$$
 (8)

and
$$I = \frac{T \ F \ V = 503 \ M \ h_2^2}{503 \ M \ h_2^2}$$
 (9)

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T " Gross weight of train in tone,

H - Munder of meters in the tening

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F V = 503 P mile pounds per hour per ton. {Y}

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The equation of the straight line BB in Fig. 2 is

$$F' = h_3 + h_A I$$
 (10)

in which

F' = Tractive effort output per motor in pounds,

I = Current per motor in amperes,

h₃ and h₄ are constants determined by the co-ordinates of any two points on the line BB.

To reduce the tractive effort to pounds per ton of gross train weight,

substitute
$$\frac{T}{M} F = F'$$
 (11)

in which

F = Tractive effort input to driving axles in pounds per ton,

T = Gross weight of train in tons,

N = Number of motors in train.

1. Optystin

Then
$$\frac{T}{H}F = h_3 + h_4I \tag{2}$$

or
$$F = \frac{M}{T} (h_3 + h_4 I)$$
, (13)

and
$$3 \cdot Operat I = \frac{T F = M h_3}{M h_4}$$
 (24)

Combining equations (9) and (14),

$$\frac{\text{T F V} - 503 \text{ M h}_{1}^{2}}{503 \text{ M h}_{2}^{2}} = \frac{\text{T F} - \text{M h}_{3}}{\text{M h}_{4}}.$$
 (15)

$$T (h_A V - 503 h_2^*) F = 503 M (h_1^* h_4 - h_2^* h_3) ,$$
 (17)

$$F = \frac{503 \text{ M} \left(\text{ h}_1^* \text{h}_4 - \text{h}_2^* \text{h}_3 \right)}{\text{T} \left(\text{ h}_4 \text{V} - 503 \text{ h}_2^* \right)}, \quad (18)$$

and
$$F = \frac{503 \text{ M}}{T} \frac{(h_1^* - h_3 h_2^* / h_4)}{(\nabla - 503 h_2^* / h_4)} . \tag{19}$$

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$$R^{\dagger} = h_{tt} + h_{t} I \tag{10}$$

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U = Eyeckive effect output per inter in possite,
I = Current per noter in expere.
h and b, are constants determined by the co-ordinate of
ent two points on the line BB.

To reduce the tractive effort to pounds par ton of gross train weight,

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in which

T - Ernotive effort liquit to driving sales in pounds per bon.
T - Gross weight of train in tous.
I - Number of motors in train.

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Letting
$$h_1 = \frac{503 \text{ M}}{T} (h_1^* - h_3 \frac{h_2^*}{h_A})$$
 (20)

and
$$h_2 = 503 \frac{h_2}{h_4}$$
 (21)

$$F = \frac{n_1}{v - h_2} \tag{22}$$

This equation (22) is the approximate formula for tractive effort F, in pounds per ton of gross train weight, in terms of the train speed V, in miles per hour, when rated voltage is applied to the terminals of the motors.

For reasons, that will appear presently, the terminal voltage of the motors is reduced below the rated voltage during certain periods in the operation of a train. In fact, there are three conditions of terminal voltage to be considered, namely:

1. Operation at rated voltage;

to alie or the train to

- 2. Operation with power shut off, that is, with zero voltage applied;
- 3. Operation with applied voltage less than rated voltage and not zero.

As has been pointed out, equation (32) applies in the first of these three cases.

while a train is moving with the power shut off, the terminal voltage, applied to the motors, is zero; hence the motor current is zero. The output of the motors however, is slightly negative. That is, a small amount of energy is extracted from the train and dissipated due to friction in the motors and gears. However the rate of extraction of this energy is so small that, for practical purposes, it is neglected so that

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This equation (22) is the approximate for analytic errors. In powers per ten of group train when the power of the train apost V.

In miles per have, when rated voltage is applied to the terminals of the motors.

For reasons, that will appear presently, the develor voltage of the motors is reduced below the revol voltage during certain periods to the operation of a train. In fact, there are three conditions of terminal voltage to be considered, namely.

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- 2. Openation with moves and off, thet in,
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While a train is moving with the power shut off, the terminal voltage, explicit to the mover, is zero; hence the motor exprest is mera. The subject of the motors however, is slightly negative. That is, a small smownt of energy is extraobed from the train and dissipated due to friction in the motors and gears. However the rate of entergiion of this owners is no mail that, for prescient purposes, it is neglected so that

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(88)

Except while starting, trains are seldom operated for any appreciable length of time with any terminal voltage less than normal applied to the motors. In special cases, a constant partial voltage may be applied. But such cases are rare and, when they must be treated, an expression for F in the form of equation (22) can readily be derived by making suitable modifications of the motor characteristic curves to accord with the partial voltage.

the electromotive force, induced in the armature, is zero. Hence, if voltage is applied to the motor terminals with the train at stand-still, the current is limited only by the resistance of the field and armature circuits combined. This resistance is necessarily so small in railway motors that, if the rated voltage were applied to the motor terminals while the train was at standstill, the motors would either be damaged seriously or develop an excessive torque causing the wheels to slip or the train to start with a severe jerk. Therefore the motor terminal voltage is varied so as to keep the current within safe limits during the starting period.

The voltage, supplied to a train, is usually constant so, in controlling continuous current series railway motors, the adjustment of motor terminal voltage is procured by connecting external resistance in series with the motors. From several points of view, the ideal control would maintain the motor current constant at its maximum permissible value throughout the starting period. For practical reasons, however, the custom is to vary the external resistance in a few steps, keeping the current within certain well defined limits while starting. Although

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The woltage, emplied to a train, is remitly constant so, in controlling continuous encreat series railway motors, the adjustment of sotor berminal veltage is produced by consecting external resistance in series with the motors. From several points of view, the ideal case twel would exist the setter oursest constant of the mentions persisting value throughout the starting period. For precitar remons, however, the custom is to very the external resistance in the winter. Inspired the custom is the very the external resistance in the starting. Inspired the current within certain well defined limits while starting. Although Although

constant current is not maintained by this mothod, it is practicable to troat the current as though it were constant at an equivalent average value. It has been shown that the motor tractive effort depends upon the motor current and is independent of the motor terminal voltage except in so far as the latter affects the magnitude of the current. Hence, an equivalent constant starting current produces a corresponding constant tractive effort. In other words, the tractive effort, F in equation (1), is treated as constant from the instant when the train begins to move until the train speed attains the value indicated in the motor characteristic curves as corresponding to the average starting current.

With the permissible average starting current fixed, the corresponding tractive effort is determined from the characteristic curves. This constant tractive effort, expressed in pounds per ton, is the value of F in equation (1) during the starting period.

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Filth the persisting structure structure oursent fixed, the deriver product tractive effect is determined from the characteristic curves. This commiss tractive effort, expressed in pounds per ten, is the value of F in equation (1) during the starting parted.

SPEED - TIME FORMULAR

A - TRAIN CROLE

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The trains of a railway system operate in what are called trips between terminals, which may be at the ends of a line of track or intermediate as at the ends of divisions. In the course of a trip, a train usually makes several steps. The period of time, that clapses between when a train leaves one stopping place and when it leaves the next, is called a train cycle.

The first event in a train cycle is the start, which is followed by successive periods of operation each involving more or less different conditions, and finally comes the stop. All the normal conditions of operation, that are not in a train cycle, may be classified under a few typical phases which are named largely in accordance with the effects that they produce upon the train speed. They are:

- 1. starting.
- 2. Accelerating,
- 3. Runing at constant speed,
- 4. Coasting,
- 5. Braking,
- 6. Stop.

Each of these phases of operation entails a unique interpretation of certain terms, particularly the input tractive effort F, in the fundamental differential equation (1). And upon these interpretations rest the solutions of this equation and thereby the formulae for the speed-time relations.

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conditions of ejeration, that are not in a train avels, may be closelfied under a few typical pands which are need largely in accordance
with the effects that they produce upon the typical apeal. They see:

- L. Stanting.
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 - S. dealths.
 - 6. Boogs

ments of these phases of operation outsits a unique interpretation of coutsin terms, particularly the imput tracelise effort V, in the france ments interpretation (1). And upon these interpretations rest the solutions of this soundies and thereby the formiles for the speed-time relations.

For obvious reasons, the brakes are not ordinarily applied while the motors are functioning and consequently, during the greater part of any train cycle,

Also curves and grades are of incidental occurrence so most train cycles include periods when

C = 0, G = 0 or C = G = 0 .

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(25)

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When B, C or G is not zero, it may be constant; but, if it is not constant, it is usually necessary and sufficiently accurate for practical purposes to treat it as constant by taking average values over definite intervals. The terms, B. C and G. will be carried as constants in the present derivations in order that the resulting formulae may be (31) universally applicable. (學+祭)-(學+V)*

Having determined the tractive effort for the three possible conditions of applied motor voltage, and having divided the train cycle into its component phases of operation, it remains to solve the above (33) equation (1) for each of these several phases.

inhogorables of this combion recovers

It has been pointed out above that, during the starting period, the voltage applied to the motor terminals is varied so as to maintain practically constant tractive effort. This permits the solution of equation (1) on the assumption that F is constant.

For obvious reasons, the brates are not ordinarily applied while the motors are functioning and consequently, suring the greater part of any train cycle.

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Also curves and grades are of incidental occurrence so more train

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Them D. G or G is not used, it may be constent; but, if it is not constant, it is not constant, it is no proceed to the mensity measurement of the first proposes to break it as comptent by taking average values over definite intervals. The terms, D. G and G. will be carried as constants in the present derivations in order that the resulting formulae may be universally applicable.

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period, the voltage applied to the motor beneficial as welled so as to period, the voltage english to the motor beneficial as varied as the motor of the complete the period the complete the complete the complete the complete.

In order to facilitate manipulation, group the constants

and, for the starting period, let

$$\alpha_{i} = 0.01(F-B-C-G-\frac{50}{\sqrt{T}})$$
, (24)

$$\beta_1 = 0.0003$$
 , (25)

$$\gamma_{i} = \frac{0.00002 \times (1 + \frac{N-1}{10})}{T} . \tag{26}$$

Then equation (1) becomes

$$\frac{dV}{dt} = \alpha, -\beta, V - \eta, V^2, \qquad (27)$$

$$dt = \frac{dV}{\alpha, -\beta, V - \gamma, V^2} , \qquad (28)$$

$$\eta_{i}dt = \frac{dV}{\frac{\alpha_{i} - \beta_{i}V - \gamma_{i}V^{2}}{\gamma_{i}}},$$
(29)

$$= \frac{dV}{\frac{\alpha_{i}}{\gamma_{i}} + \frac{\beta_{i}^{2}}{4\gamma_{i}^{2}} - \frac{\beta_{i}}{4\gamma_{i}^{2}} - \frac{\beta_{i}}{N}V - V^{2}}$$

$$(30)$$

$$= \frac{dV}{\left(\frac{\alpha_i}{\gamma_i} + \frac{A^2}{4\gamma^2}\right) - \left(\frac{A_i}{2\gamma_i} + V\right)^2} \quad . \tag{31}$$

(40)

Since $\frac{\beta_i}{2\gamma_i}$ is constant,

F MESER por Rour, is

$$d\left(\frac{B}{2T} + V\right) = dV \tag{32}$$

so that

$$\gamma_{i}dt = \frac{d\left(\frac{\partial z_{i}}{\partial y_{i}} + V\right)}{\left(\frac{\partial z_{i}}{\partial y_{i}} + \frac{\partial z_{i}}{\partial y_{i}^{2}}\right) - \left(\frac{\partial z_{i}}{\partial y_{i}} + V\right)^{2}} \tag{33}$$

Integration of this equation renders

$$7,t = \frac{1}{2\sqrt{\frac{\alpha_{i}}{7_{i}} + \frac{\beta_{i}^{2}}{47_{i}^{2}}}} \log_{2}\left[\frac{\sqrt{\frac{\alpha_{i}}{7_{i}} + \frac{\beta_{i}^{2}}{47_{i}^{2}} + (\frac{\beta_{i}}{27_{i}} + V)}}{\sqrt{\frac{\alpha_{i}}{7_{i}} + \frac{\beta_{i}^{2}}{47_{i}^{2}} - (\frac{\beta_{i}}{27_{i}} + V)}}\right] + C_{i}'$$
(34)

where C_i is a constant of integration. Dividing both members of equation (54) by %,

$$t = \frac{1}{\sqrt{4\alpha_1 7_1 + \beta_1^2}} \log_e \left[\frac{\sqrt{4\alpha_1 7_1 + \beta_1^2} + \beta_1 + 27_1 V}{\sqrt{4\alpha_1 7_1 + \beta_1^2} - \beta_1 - 27_1 V} \right] + \frac{C_1'}{\gamma_1'}$$
(35)

Transforming to common logarithms in order to facilitate computation,

$$t = \frac{2.30}{\sqrt{4\alpha_i 1_i + \beta_i^2}} \log_{10} \left[\frac{\sqrt{4\alpha_i 1_i + \beta_i^2 + \beta_i + 21_i V}}{\sqrt{4\alpha_i 1_i + \beta_i^2 - \beta_i - 21_i V}} + \frac{C_i'}{1_i} \right]$$
(36)

In order to Assiltante manipulation, group the constants

del the tee desting period, let

$$\rho_{\rm s} = 0.0003$$
(25)

$$\gamma_{i} = \frac{0.00002 \times (1 + N - 1)}{70}$$
. (26)

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$$\frac{dV}{dt} = \alpha_{r} - \beta_{r}V - \gamma_{r}V^{2}, \qquad (27)$$

$$dt = \frac{dV}{\alpha_s - \beta_s V - \gamma_s V^2} \,, \tag{28}$$

$$T_i dt = \frac{dV}{\frac{m_i - \frac{m_i}{2}V - V^2}{2}} \tag{29}$$

(31)
$$\frac{\sqrt{b}}{(V+\frac{2}{2N^2})-(\frac{2}{2N}+\frac{2}{2N})} = \frac{1}{2N^2}$$

Since 2% is constant.

$$d\left(\frac{B}{Z_{ij}^{T}}+V\right)=dV \tag{32}$$

$$7dt = \frac{d(\frac{d}{dt} + \frac{d}{dt})}{(\frac{d}{dt} + \frac{d}{dt})} = 3dt$$
(33)

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where C'is a constant of integration. Dividing both members of squarties (SA) by Z.

$$t = \frac{1}{\sqrt{4\alpha_i l_i + \beta_i^2} + \beta_i + 2l_i V} + \frac{C}{l_i}$$
 (35)

Transforming to common logarithms in order to facilitate common of

$$t = \frac{2.30}{\sqrt{400,11+10^2}} \log_{10} \left[\frac{\sqrt{400,11+10^2+10,1+21,V}}{\sqrt{400,11+10^2-10,1-21,V}} + \frac{C!}{2!} \right]$$
(36)

If the train speed is V_0 at the instant t_0 ; the time t_0 , in the starting period, at which the speed will reach any other value V_0 is given by

$$t-t_{o} = \frac{2.30}{\sqrt{4\alpha,7,+\beta_{i}^{2}}} log_{vo} \left[\frac{\sqrt{4\alpha,7,+\beta_{i}^{2}} + \beta_{i} + 27,V}{\sqrt{4\alpha,7,+\beta_{i}^{2}} - \beta_{i} - 27,V} \right] - \frac{2.30}{\sqrt{4\alpha,7,+\beta_{i}^{2}}} log_{vo} \left[\frac{\sqrt{4\alpha,7,+\beta_{i}^{2}} + \beta_{i} + 27,V_{o}}{\sqrt{4\alpha,7,+\beta_{i}^{2}} - \beta_{i} - 27,V_{o}} \right] ; \quad (37)$$

that is, ______ log_(zx,-B,V+V-fax;+B) - log_(zx,-B,V-V-fax;+B)

$$t = t_o + \frac{2.30}{\sqrt{4\alpha_1 7_1 + \beta_2^2}} \left\{ log_{ro} \left[\frac{\sqrt{4\alpha_1 7_1 + \beta_2^2 + \beta_1 + 2\gamma_1 V}}{\sqrt{4\alpha_1 7_1 + \beta_2^2 - \beta_1 - 2\gamma_1 V}} \right] - log_{ro} \left[\frac{\sqrt{4\alpha_1 7_1 + \beta_2^2 + \beta_1 + 2\gamma_1 V_0}}{\sqrt{4\alpha_1 7_1 + \beta_2^2 - \beta_1 - 2\gamma_1 V_0}} \right] \right\}$$
(38)

 $t = t_{o} + \frac{2.30}{\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}}} \left[log_{io} \left(\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}} + \beta_{i} + 27_{i}V_{i} \right) - log_{io} \left(\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}} - \beta_{i} - 27_{i}V_{i} \right) \right]$ $- log_{io} \left(\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}} + \beta_{i} + 27_{i}V_{o} \right) + log_{io} \left(\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}} - \beta_{i} - 27_{i}V_{o} \right)$ (39)

where t and to are expressed in seconds and V and Vo in miles per hour-Combining equations (36) and (38), and letting

$$C_{i} = \frac{C_{i}}{\gamma_{i}} \tag{40}$$

(35)

(43)

it is seen that

$$C_{i} = \frac{C'_{i}}{7_{i}} = t_{o} - \frac{2.30}{\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}}} \log_{10} \left[\frac{\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}} + \beta_{i} + 2\gamma_{i}V_{o}}{\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}} - \beta_{i} - 2\gamma_{i}V_{o}} \right]$$
(41)

line than full speed in cotained. Also, in subco-

$$C_{r} = t_{o} + \frac{2.30}{\sqrt{4\alpha_{r}\eta_{r} + \beta_{r}^{2}}} \left[log_{ro} \left(\sqrt{4\alpha_{r}\eta_{r} + \beta_{r}^{2}} - \beta_{r} - 2\gamma_{r}V_{o} \right) - log_{ro} \left(\sqrt{4\alpha_{r}\eta_{r} + \beta_{r}^{2}} + \beta_{r} + 2\gamma_{r}V_{o} \right) \right]$$
(42)

and equation (39) reduces to

$$t = C_{1} + \frac{2.30}{\sqrt{4\alpha_{1}7_{1} + \beta_{1}^{2}}} \left[log_{\infty} \left(\sqrt{4\alpha_{1}7_{1} + \beta_{1}^{2}} + \beta_{1} + 2\gamma_{1}V \right) - log_{\infty} \left(\sqrt{4\alpha_{1}7_{1} + \beta_{1}^{2}} - \beta_{1} - 2\gamma_{1}V \right) \right]. (43)$$

Special Case: Vo = 0 , to = 0 (5 - 6 - 5)

If a train is started from standstill and time is measured from the instant of starting, so that $V_0 = 0$ and $t_0 = 0$; equation (38) shows that the number of seconds, required for the train to attain the speed V miles per hour, is

If the train eyout is Yo at the instant to the time the time to in the the the the the the the starting parted, at which the speed will read any other value V. is given by

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$$t = t_0 + \frac{2.30}{\sqrt{4\alpha_0} 1 + \beta_0^2} \left[log_0 \left[\frac{\sqrt{4\alpha_0} 1 + \beta_0^2 + \beta_0 + 2\pi V}{\sqrt{4\alpha_0} 1 + \beta_0^2} - log_0 \left[\frac{\sqrt{4\alpha_0} 1 + \beta_0^2 + \beta_0 + 2\pi W}{\sqrt{4\alpha_0} 1 + \beta_0^2} \right] \right]$$
 (38)

70

$$t = t_o + \frac{2.30}{\sqrt{4a_i t_i^2 + \beta_i^2}} \left[log_{io} \left(\sqrt{4a_i t_i^2 + \beta_i^2 + 2t_i t} \right) - log_{io} \left(\sqrt{4a_i t_i^2 + \beta_i^2} - \beta_i - 2t_i t t \right) \right] \\ - log_{io} \left(\sqrt{4a_i t_i^2 + \beta_i^2} + \beta_i + 2t_i t_i \right) + log_{io} \left(\sqrt{4a_i t_i^2 + \beta_i^2} - \beta_i - 2t_i t_i \right) \right]$$

where t and to are appreced in seconds and V and Vo in miles per hour.
Combining equations (56) and (58), and letting

$$c_i = \frac{c_i}{2}$$
 (40)

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$$C_{i} = \frac{C'_{i}}{N} = t_{o} - \frac{230}{\sqrt{44\pi/1+3}} log_{o} \left[\frac{\sqrt{43\pi/1+6^{2}} + \rho_{i} + 2\pi/16}{\sqrt{43\pi/1+6^{2}} - \rho_{i} - 2\pi/16} \right] \tag{41}$$

70

$$C_{i} = t_{i} + \frac{2.30}{\sqrt{4x_{i}^{2} + \beta_{i}^{2}}} \left[log_{i} \left(\sqrt{4x_{i}^{2} + \beta_{i}^{2}} + \beta_{i} - 2x_{i}^{2} \right) - log_{i} \left(\sqrt{4x_{i}^{2} + \beta_{i}^{2}} + \beta_{i} + 2x_{i}^{2} \right) \right]$$

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Special desc; You of to wo

If a tenin is strated from Sound in and the is sentent of a society (88) for the instant of strategy, so that We was the train to extain the short what the nuclear of senguine, required for the train to extain the space V miles year house in

$$t_{s} = \frac{2.30}{\sqrt{4\alpha,1,+\beta_{s}^{2}}} \log_{10} \left[\frac{\sqrt{4\alpha,1,+\beta_{s}^{2}+\beta_{s}+2\gamma,V}}{\sqrt{4\alpha,1,+\beta_{s}^{2}-\beta_{s}-2\gamma,V}} \right] - \frac{2.30}{\sqrt{4\alpha,1,+\beta_{s}^{2}}} \log_{10} \left[\frac{\sqrt{4\alpha,1,+\beta_{s}^{2}+\beta_{s}}}{\sqrt{4\alpha,1,+\beta_{s}^{2}-\beta_{s}}} \right]$$
(44)

$$=\frac{2.30}{\sqrt{4\alpha,1,+\beta_{i}^{2}}}\log_{10}\left[\frac{(\sqrt{4\alpha,1,+\beta_{i}^{2}}+\beta_{i}+21,V)(\sqrt{4\alpha,1,+\beta_{i}^{2}}-\beta_{i})}{(\sqrt{4\alpha,1,+\beta_{i}^{2}}-\beta_{i}-21,V)(\sqrt{4\alpha,1,+\beta_{i}^{2}}+\beta_{i})}\right]$$
(45)

$$= \frac{2.30}{\sqrt{4\alpha_i 7_i + \beta_i^2}} \log_{10} \left[\frac{2\alpha_i - \beta_i V + V \sqrt{4\alpha_i 7_i + \beta_i^2}}{2\alpha_i - \beta_i V - V \sqrt{4\alpha_i 7_i + \beta_i^2}} \right]$$

$$(46)$$

$$t_{s} = \frac{2.30}{\sqrt{4\alpha_{s}7_{s}^{2}+\beta_{s}^{2}}} \left[log_{10}(2\alpha_{s}-\beta_{s}V+V\sqrt{4\alpha_{s}7_{s}^{2}}+\beta_{s}^{2}) - log_{10}(2\alpha_{s}-\beta_{s}V-V\sqrt{4\alpha_{s}7_{s}^{2}}+\beta_{s}^{2}) \right]$$
(47)

C - NORMAL TENNINAL VOLTAGE APPLIED TO MOTORS

B = 0.01 0.03h - (8+C+G+ 50).

At the conclusion of the above starting period, when full rated voltage is applied to the motor terminals, the motor proceeds to function in accordance with its characteristic curves. Ordinarily, there will be a period of acceleration between the end of the starting period and the time when full speed is attained. Also, in subsequent periods of a train cycle, there may be acceleration, positive or negative, as grades, curves, etc. are encountered. As long as rated terminal voltage is applied to the motors, the relation of tractive effort and speed is approximately

$$F = \frac{h_i}{V - h_2} \quad . \tag{22}$$

(34)

Substituting this in equation (1) gives

$$\frac{dV}{dt} = 0.01 \left[\frac{h_0}{V - h_0} - B - C - G - \frac{50}{\sqrt{T}} - 0.03V - \frac{0.002 \times (1 + \frac{N - I}{IO})V^2}{T} \right]$$
(48)

$$= \frac{O.01}{V - h_2} \begin{cases} h_1 + h_2(B + C + G + \frac{50}{\sqrt{T}}) \\ + \left[0.03 h_2 - (B + C + G + \frac{50}{\sqrt{T}}) \right] V \\ + \left[\frac{0.002X}{T} \left(1 + \frac{N - I}{I0} \right) h_2 - 0.03 \right] V^2 \\ + \left[\frac{-0.002X}{T} \left(1 + \frac{N - I}{I0} \right) \right] V^3 \end{cases}$$

$$(49)$$

$$t_{6} = \frac{z.30}{\sqrt{4\alpha R + \beta^{2}}} log_{10} \left[\frac{\sqrt{4\alpha R + \beta^{2} + \beta_{1} + \beta^{2} + \beta_{1} + \beta^{2} + \beta_{1}}}{\sqrt{4\alpha R + \beta^{2}}} log_{10} \left[\frac{\sqrt{4\alpha R + \beta^{2} + \beta_{1} + \beta^{2} + \beta^{$$

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Ab the conclusion of the starting pariot, when full rated voltage is more than full rated voltage is more to the motion terminal, the motion of contrast orders. Ordinarily, to function in described with its described contrast. Ordinarily, there will be a period of acquiration between the end of the start. Inc, is subserting period and the time when full appear is attained. Also, is subsert quant periods of a train eyele, there may be acquirabled, positive or negative, as grader, convex, etc encountered. As long as trained to the acquiration of the approximately.

$$F = \frac{h_1}{V - h_2} .$$

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$$\frac{dV}{dt} = 0.0i \left[\frac{h_1}{V - h_2} - B - C - G - \frac{50}{\sqrt{T}} - 0.09V - \frac{0.002 \times (1 + \frac{N-1}{10})V^2}{10} \right]$$

$$= \frac{o \cdot o \cdot o}{V - h_{z}} \left\{ (B + C + 6 + \frac{5 \circ}{\sqrt{7}}) \right\} V$$

$$= \frac{o \cdot o \cdot o}{V - h_{z}} \left\{ (A + C + 6 + \frac{5 \circ}{\sqrt{7}}) \right\} V^{z}$$

$$+ \left[\frac{o \cdot o \circ z \times (A + \frac{N-1}{\sqrt{7}}) h_{z} - o \cdot o \cdot o}{V - h_{z}} \right] V^{z}$$

$$+ \left[\frac{-o \cdot o \circ z \times (A + \frac{N-1}{\sqrt{7}})}{V - A + \frac{N-1}{\sqrt{7}}} \right] V^{z}$$

$$(4.5)$$

(ss)

(48)

Let
$$\delta_2 = 0.01 \left[\frac{-0.002 \times (1 + \frac{N-1}{10})}{7} \right]$$
 (50)

$$\frac{dV}{dt} = \frac{\delta_z}{V - h_2} \left\{ \frac{0.01}{\delta_z} \left[h_1 + h_2 (B + C + G + \frac{50}{\sqrt{7}}) \right] + \frac{0.01}{\delta_z} \left[0.03 h_2 - (B + C + G + \frac{50}{\sqrt{7}}) \right] V + \frac{0.01}{\delta_z} \left[\frac{0.002 \, X}{7} \left(1 + \frac{N - 1}{10} \right) h_2 - 0.03 \right] V^2 + V^3 \right\} . \tag{51}$$

Let
$$\alpha_z = \frac{O.01}{\delta_2} \left[h_1 + h_2 \left(B + C + G + \frac{50}{\sqrt{T}} \right) \right] , \qquad (52)$$

$$\beta_2 = \frac{0.01}{\delta_2} \left[0.03 h_2 - (B + C + G + \frac{50}{\sqrt{7}}) \right], \qquad (53)$$

$$\gamma_2 = \frac{0.01}{\delta_2} \left[\frac{0.002X}{T} \left(1 + \frac{N-1}{10} \right) h_2 - 0.03 \right] .$$
 (54)

(63)

(54)

(68)

(69)

Then
$$\frac{dV}{dt} = \frac{\delta_z}{V - h_z} \left(\alpha_z + \beta_z V + \gamma_z V^2 + V^3 \right) \qquad (55)$$

181° + 984+ 1 = 0

1 - Balancing Speed

In railway parlence, a train operating at constant speed with rated voltage applied to the motor terminals is spoken of as ruming at balancing speed. The term arises from the fact that when a train is operating at constant speed, the power imput is just balanced by the power dissipated plus the rate of storage of potential energy. In other words, there is no kinetic energy being stored in, or extracted from, the train.

Since the speed, V in the foregoing equations, becomes constant at the balancing speed, the latter can be designated as a particular value of V, say V2.

Also, when the speed is constant, the rate of change of speed is zero; that is, when $V = V_2$, $\frac{dV}{dt} = 0$. (56)

- 25 -

W = - F + V + + 27 + - F - V + + 27 10

$$\mathcal{E}_{2} = 0.01 \left[\frac{-0.002 \times (1 + \frac{N-1}{10})}{7} \right].$$

$$\frac{dV}{dt} = \frac{\delta_{2}}{V - h_{2}} \left\{ \frac{aot}{\delta_{1}} \left[\frac{h}{h} + h_{2} (\theta + C + G + \frac{co}{2}) \right] + \frac{co}{\delta_{2}} \left[aosh_{1} - (\theta + C + G + \frac{co}{2}) \right] V \right\}$$

$$+ \frac{co}{\delta_{2}} \left[\frac{coocs}{\delta_{1}} \left[\frac{h}{h} + \frac{h}{h} \left[h + \frac{h}{h} \right] h_{2} - aos \right] V^{2} + V^{3} \right]$$

(50

(51

(52

(53

(54

(3C

Let
$$\alpha_{z} = \frac{o.ot}{\delta_{z}} \left[h_{t} + h_{z} \left(B + C + G + \frac{SQ}{s} \right) \right] ,$$

$$\beta_{z} = \frac{o.ot}{\delta_{z}} \left[o.o3h_{z} - \left(B + C + G + \frac{SQ}{s} \right) \right] ,$$

$$\gamma_2 = \frac{o.o.}{\delta_z} \left[\frac{o.oo_2 X}{T} (1 + \frac{N-1}{10}) h_z - o.o. \right]$$

$$\frac{dV}{dt} = \frac{\delta_{s}}{V - h_{s}} (\alpha_{s} + \beta_{s} V + \beta_{s} V^{2} + V^{3}) .$$

1 - Balancing coool

In realization applied to the motor berminals is special escal with rated rollings applied to the motor berminals is special of as running at believing upoels. The term arises from the fact that and the free that the fact is just believed by the power important appeal, the power imput is just believed by the power dissipated plus the rate of aborage of potential, and content on minute company being stored in.

-not sended the foreign and foreign advertised and the death white can be designabled as a castle of the latter and of the latter and of the latter and the

to sero; the when V " Vo W = 0.

Then, from equation (55),

$$\frac{\delta_2}{V_2 - h_2} \left(\alpha_2 + \beta_2 V_2 + \gamma_2 V_2^2 + V_2^3 \right) = 0 . \tag{57}$$

Dividing this by $\frac{\delta_z}{V_z - h_z}$,

$$V_z^3 + \frac{1}{2}V_2^2 + \beta_2 V_2 + \alpha_2 = 0 . \qquad (58)$$

In order to determine the value of V2 .

1et
$$V_2 = W - \frac{7_2}{3}$$
 (59)

Then
$$W^3 + (\beta_2 - \frac{\gamma_2^2}{3})W + (\alpha_2 - \frac{\beta_2\gamma_2}{3} + \frac{2\gamma_2^3}{27}) = 0$$
. (60)

Let
$$q = \beta_2 - \frac{\gamma_2^2}{3} \tag{61}$$

and
$$r = \alpha_2 - \frac{\beta_1 \gamma_2}{3} + \frac{2 \gamma_1^3}{27}$$
. (62)

Then
$$W^3 + qW + r = 0 (63)$$

Let
$$W = y + 3$$
 where $y = \frac{-q}{33}$; (64)

that is, let
$$W = 3 - \frac{9}{33} . \tag{65}$$

Then
$$(3-\frac{q}{33})^3 + q(3-\frac{q}{33}) + r = 0$$
 (66)

or
$$3^3 - 93 + \frac{9^2}{35} - \frac{9^3}{273^3} + 93 - \frac{9^2}{35} + r = 0$$
; (67)

that is,
$$3^3 + r - \frac{q^3}{273} = 0$$
, (68)

and
$$3^6 + r 3^3 - \frac{q^3}{27} = 0$$
, (69)

whence
$$3^3 = -\frac{r}{2} \pm \sqrt{\frac{r^2}{4} + \frac{q^3}{27}}$$
 (70)

Now
$$y^3 = \frac{-q^3}{273^3} = -\frac{r}{2} \mp \sqrt{\frac{r^2}{4} + \frac{q^3}{27}}$$
, (71)

so that
$$y + 3 = \left[-\frac{r}{2} \pm \sqrt{\frac{r^2 + 9^3}{4}} \right]^{1/3} + \left[-\frac{r}{2} \mp \sqrt{\frac{r^2 + 9^3}{4}} \right]^{1/3} \tag{72}$$

or, by (64),
$$W = \left[-\frac{r}{2} + \sqrt{\frac{r^2 + q^3}{4^2 + 27}} \right]^{\frac{r}{3}} + \left[-\frac{r}{2} - \sqrt{\frac{r^2 + q^3}{4^2 + 27}} \right]^{\frac{r}{3}} . (73)$$

men, from equation (UE).

$$\frac{\delta_{z}}{V_{2}-h_{R}}(\alpha_{z}+\beta_{z}V_{z}+\beta_{z}V_{z}^{2}+V_{z}^{3})=0.$$

(57)

(58)

(59)

(09)

Dividing this by $\frac{\delta_z}{V-h}$.

$$V_{\mu}^{3} + \gamma_{\mu}^{2} V_{\mu}^{2} + \beta_{\mu} V_{\mu} + \alpha_{\mu} = 0$$
.

in order to determine the value of Vg :

$$V_{i}=W-\frac{Z_{i}}{3}.$$

Then
$$W^3 + (\rho_2 - \frac{\gamma_2}{3})W + (\alpha_2 - \frac{\rho_2\gamma_2}{3} + \frac{\rho_2\gamma_2}{2\gamma_1}) = 0$$
.

$$Q = \beta_2 - \underline{Z}^2 \tag{61}$$

$$Q = B^2 - \frac{3}{2}$$

and
$$r = \alpha_1 - \frac{6}{3} + \frac{2\pi^2}{2}$$
. (62)

Them
$$W^3 + qW + r = 0 . (63)$$

Let
$$W = y + 3$$
 where $y = \frac{-2}{3y}$; (64)

where to, low
$$M = 3 - \frac{3}{3}$$
. (65)

(3-
$$\frac{g}{33}$$
)³ + $g(3-\frac{g}{33})$ + $r = 0$ (66)

$$3^{3} - 93 + 3^{3} - \frac{2^{3}}{3^{3}} + 93 - \frac{2^{3}}{3^{3}} + r = 0 ; (67)$$

that to,
$$3^2 + r - \frac{q^2}{273} = 0$$
, (68)

(69)
$$g_{+} = \frac{42}{3} - \frac{43}{5} = 0$$

whence
$$y^3 = -\frac{c}{2} \pm \sqrt{\frac{c}{4} + \frac{c}{42}}$$
, (70)

$$y^{2} = \frac{-q^{2}}{273^{2}} = -\frac{7}{4} + \frac{7}{27},$$
 wow

80 Week
$$y + 3 = \left[-\frac{5}{5} \pm \sqrt{\frac{5}{4}} + \frac{7}{52} \right]_{p} + \left[-\frac{5}{5} \pm \sqrt{\frac{4}{4}} + \frac{5}{52} \right]_{p} = 5 + 5$$

or, by (64),
$$W = \begin{bmatrix} -2 + \sqrt{\frac{2}{4} + \frac{2}{27}} \\ -2 + \sqrt{\frac{2}{4} + \frac{2}{27}} \end{bmatrix} + \begin{bmatrix} -2 - \sqrt{\frac{2}{4} + \frac{2}{27}} \\ -2 - \sqrt{\frac{2}{4} + \frac{2}{27}} \end{bmatrix}^{1/2}$$
. (73)

In general, W will have three distinct values, at least one of which is real. That is.

$$W^3 + qW + r = 0 (63)$$

has three roots, and at least one root is real.

Let m be a real value of

$$\left[-\frac{r}{z} + \sqrt{\frac{r^2}{4} + \frac{g^3}{27}}\right]^{1/3}, \qquad (74)$$

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and let n be a real value of
$$\left[-\frac{r}{2} - \sqrt{\frac{r^2 + \frac{q^3}{27}}{4^2 + \frac{q^3}{27}}} \right]^{\frac{1}{3}}.$$
 (75)

Then m + n will be a real root of W.

Let
$$\omega = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$
, $\omega^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$, $j = +\sqrt{-1}$. (76)

The the roots of
$$W^3 + qW + r = 0$$
 (63)

are
$$m+n$$
, $\omega m+\omega^2 n$, $\omega^2 m+\omega n$. (77)

And, since
$$V_2 = W - \frac{\gamma_2}{3}$$
, (59)

the values of
$$V_2$$
 in $V_z^3 + 7_z V_z^2 + \beta_z V_z + \alpha_z = 0$ (58)

$$\rho_{i} = -\frac{J_{2}}{3} + m + n \quad , \tag{78}$$

$$\rho_2 = -\frac{\gamma_2}{3} + \omega m + \omega^2 n , \qquad (79)$$

$$\rho_3 = -\frac{\gamma_2}{3} + \omega^2 m + \omega n \quad . \tag{80}$$

It is seen, from equation (75) and the roots (77), that,

It is seen, from equation (75) and the roots (77), that,
$$\frac{r^2}{4} + \frac{q^3}{27} = 0, \qquad (81)$$

$$m = n \qquad (82)$$

$$m = n \tag{82}$$

(20)

and the values of W are
$$2m$$
, $-m$, $-m$, all real. (83)

That is, all the values of W, and consequently of V2 would be real

one funct to general, W will have three distinct values, at least or or valid to to

$$W^3 + qW + r = 0 (63)$$

has three roots, and at least one root to real.

lot a bu a real value of

to enlay leer a ed n tel has

$$\left[-\frac{c}{2} - \sqrt{\frac{c^2 + \frac{2c}{2}}{27}} \right]^{16}, \tag{75}$$

(74)

.W to doon lear a ed Lilw a * m nor?

Int
$$\omega = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$
, $\omega^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$, $j = +\sqrt{-1}$. (76)

The tops of
$$M^3 + q M + r = 0$$
 (63)

ore
$$m+n$$
, $\omega m+\omega^2 n$, $\omega^2 m+\omega n$. (77)

And, since
$$V_2 = W - \frac{\gamma_2}{3}$$
, (39)

we written of
$$\Lambda^S$$
 in $\Lambda^S_2 + \lambda^2 \Lambda^S_2 + \lambda^2 \Lambda^2 + \alpha^2 = 0$ (38)

$$\rho_{s} = -\frac{t_{s}}{s} + m + n \quad , \tag{78}$$

$$\rho_{\alpha} = -\frac{\gamma_{\alpha}}{3} + \omega m + \omega^{2} n , \qquad (79)$$

$$\rho_3 = -\frac{\gamma_1}{3} + \omega^2 m + \omega n . \qquad (80)$$

It is seen, from squablem (WY) and the woods (WY), that,

$$\frac{r^2}{4} + \frac{q^2}{27} = 0 , \qquad (8i)$$

$$m = n$$
 (92)

and the values of W are $2m_s-m_s$, will real. (85) That is, all the values of W, and consequently of V_0 would be real

which might lead to ambiguity. However, in the problem at hand, an investigation of the ranges of values of

$$q = \beta_z - \frac{\gamma_z^2}{3} \tag{61}$$

(05)

(190)

(811)

(54)

and

$$r = \alpha_2 - \frac{\beta_2 \gamma_2}{3} + \frac{\gamma_2^3}{27} \tag{62}$$

shows that, in ordinary cases.

$$\frac{r^2}{4} + \frac{9^3}{27} \neq 0 . (84)$$

In other words, the solution yields only one real value of v_2 and two imaginary values. The real, or principal, value of V2 is the actual balancing speed.

a(10) = (V-12)(V-12)(V-12) = V2+12V+12V+12

2 - Acceleration

The next step is to derive a formula by which the speed-time relations can be determined for periods when the speed of the train is increasing or decreasing with rated terminal voltage applied to the motors.

In the preceding section, the fundamental equation (1) was adapted to the condition for full voltage applied to the motor terminals, (02) and reduced to simplest terms, namely:

$$\frac{dV}{dt} = \frac{\delta_2}{V - h_2} (\alpha_2 + \beta_2 V + \gamma_2 V^2 + V^3) . \tag{55}$$

Then it was shown that, when

either
$$V_2 = \rho_1 = -\frac{\gamma_2}{3} + m + n$$
, (78)

$$V_2 = \rho_2 = -\frac{7_2}{3} + \omega m + \omega^2 n , \qquad (79)$$

or
$$V_2 = \rho_3 = -\frac{\gamma_2}{3} + \omega^2 m + \omega n \tag{80}$$

were substituted for V in
$$V^3 + \frac{1}{2}V^2 + \frac{1}{3}V + \frac{1}{4}V^2 + \frac{1}{3}V + \frac{1}{4}V^2 + \frac{1}{3}V + \frac{1}{4}V^2 + \frac{1}{3}V^2 + \frac{1$$

$$V_{2}^{3} + V_{2}V_{1}^{2} + \beta_{2}V_{2} + \alpha_{2} = 0 . {(58)}$$

ns thank to meldong out al , wrough a valuation of hael thain chick to enview to segmen out to nelvegideowni

above that, is emitted trace, $\frac{r^2+\frac{q^2}{27}}{2} \neq 0$.

In other words, the columbia yields only me real value of Y, and two inner values. The real, or principal, value of Vy is the actual .looge gaiocaled

2 - Accolemition - 2

The next step is to derive a formula by which the speed-bise relations can be determined for portods when the egeod of the train is off of heliges against family of house will guinesupan to guinesupal * # 10 \$ OM

In the preceding section, the fractionaries equation (1) was . alanimes when shi of helique egation limit were meldibeco one of hedgans inflower tendent teelquie of hecomes has

$$\frac{dV}{dt} = \frac{\delta_1}{V - h_2} (\alpha_2 + \beta_2 V + 7_2 V^2 + V^3) . \tag{35}$$

(84)

made alone dead that's when

either
$$V_2 = \rho_1 = -\frac{\gamma_2}{3} + m + n$$
, (78)

$$V_2 = \rho_2 = -\frac{\tau_2}{3} + \omega m + \omega^2 n$$
, (79)

or
$$V_2 = \rho_3 = -\frac{f_2}{3} + \omega^3 m + \omega n \tag{80}$$

mere enhabiteded for V in

$$V_2^2 + v_1 V_2^2 + \alpha_2 V_2 + \alpha_3 = 0 . (58)$$

Therefore

$$V^{3} + \frac{1}{2}V^{2} + \beta_{2}V + \alpha_{2} = (V - \rho_{1})(V - \rho_{2})(V - \rho_{3}). \tag{85}$$

Also
$$\alpha_2 = -\rho_1 \rho_2 \rho_3 \quad , \tag{86}$$

$$\beta_2 = \rho_1 \rho_2 + \rho_2 \rho_3 + \rho_3 \rho_1 , \qquad (87)$$

= 4-log (V-p) + 4-log (V-p) + 1-log (V-p) + 5. (101)

$$\gamma_2 = -(\rho_1 + \rho_2 + \rho_3)$$
 (88)

Introducing the relation (85) into equation (55) renders

$$\frac{dV}{dt} = \frac{\delta_2}{V - h_2} (V - \rho_1)(V - \rho_2)(V - \rho_3) \tag{89}$$

which transposed is
$$dt.\delta_2 = \frac{(V-h_2)dV}{(V-\rho_2)(V-\rho_2)}.$$
 (90)

Let
$$f(V) = V - h_2 \tag{91}$$

and
$$g(V) = (V-\rho_2)(V-\rho_2)(V-\rho_3) = V^3 + \frac{1}{2}V^2 + \frac{1}{2}V + \alpha_2$$
, (92)

whence
$$g'(V) = \frac{dg}{dV} = 3V^2 + 27_2V + \beta_2$$
 (93)

That is,
$$\frac{f(V)}{g(V)} = \frac{V - h_2}{(V - \rho_1)(V - \rho_2)(V - \rho_3)}$$
, (94)

which may be resolved into partial fractions, thus

$$\frac{f(V)}{g(V)} = \frac{l_2}{V - \rho_1} + \frac{m_2}{V - \rho_2} + \frac{n_2}{V - \rho_3} , \qquad (95)$$

(105)

where

$$l_{2} = \frac{f(\rho_{i})}{g'(\rho_{i})} = \frac{\rho_{i} - h_{2}}{3\rho_{i}^{2} + 2\gamma_{2}\rho_{i} + \beta_{2}} , \qquad (96)$$

$$m_2 = \frac{f(\beta_2)}{g'(\rho_2)} = \frac{\rho_2 - h_2}{3\rho_1^2 + 2\gamma_2 \rho_2 + \beta_2} , \qquad (97)$$

$$n_2 = \frac{f(\rho_2)}{g'(\rho_3)} = \frac{\rho_3 - h_2}{3\rho_3^2 + 2\eta_2\rho_3 + \beta_2} \qquad (98)$$

From the equations (90), (94) and (95), it is seen that

$$dt.\delta_{2} = \frac{l_{2}dV}{V - \rho_{1}} + \frac{m_{2}dV}{V - \rho_{2}} + \frac{n_{2}dV}{V - \rho_{3}}.$$
 (99)

Integration then renders

$$t.\delta_2 = l_2 \log_e(V-\rho_1) + m_2 \log_e(V-\rho_2) + n_2 \log_e(V-\rho_3) + C_2' (100)$$

where Co is the constant of integration.

- explored

$$V^2 + 7_5 V^2 + \Omega_2 V + \alpha_n = (V - \rho_1)(V - \rho_2)(V - \rho_3)$$
 (85)

Also
$$\alpha_2 = -\rho_1 \rho_2 \rho_3$$
, (86)

$$\beta_k = \rho_i \rho_t + \rho_t \rho_3 + \rho_5 \rho_i , \qquad (87)$$

$$I_{2} = -(\rho_{1} + \rho_{2} + \rho_{3})$$
 (88)

Introducing the relation (85) into oquation (85) rendere

$$\frac{dV}{dt} = \frac{\delta_{v}}{V - h_{z}} (V - \rho_{z}) (V - \rho_{z}) (V - \rho_{z})$$
 (89)

which transposed is
$$dt.S_a = \frac{(V-h_a)dV}{(V-\rho_a)(V-\rho_a)}$$
. (90)

Let
$$f(V) = V - h_z$$
 (91)

and
$$g(V) = (V-\rho_3)(V-\rho_3) = V^3 + 7_3 V^2 + \rho_3 V + \alpha_3$$
, (92)

whence
$$g'(V) = \frac{dg}{dV} = 3V^2 + 2J_2V + \beta_2$$
 (93)

Then in
$$\frac{f(V)}{g(V)} = \frac{V - h_a}{(V - \rho_a)(V - \rho_a)(V - \rho_a)}$$
 (94)

which may be restled into purital fractions, then

$$\frac{f(V)}{g(V)} = \frac{I_{c}}{V - \rho_{c}} + \frac{m_{s}}{V - \rho_{d}} + \frac{\eta_{2}}{V - \rho_{d}}, \qquad (35)$$

where
$$I_2 = \frac{f(\rho_i)}{g'(\rho_i)} = \frac{\rho_i - h_2}{g(\rho_i)} = \frac{1}{g(\rho_i)} = \frac{1}{g(\rho$$

$$m_s = \frac{F(q_s)}{Q(p_s)} = \frac{p_s - h_s}{3p_s^2 + 2h_s p_s + p_s}$$
, (97)

$$n_2 = \frac{f(\rho_3)}{g'(\rho_3)} = \frac{g\rho_3 + 2f_1\rho_3 + \rho_4}{g\rho_3 + 2f_1\rho_3 + \rho_4}$$
 (96)

From the equations (90), (94) and (96), it is seen that

$$dt.\delta_z = \frac{l_z dV}{V - \rho_z} + \frac{m_z dV}{V - \rho_z} + \frac{n_z dV}{V - \rho_z} . \tag{39}$$

Integration that renders

$$t.\delta_2 = l_2 \log_2(V - \rho_1) + m_2 \log_2(V - \rho_2) + m_2 \log_2(V - \rho_3) + C_2' (100)$$

where do is the consecut of integration.

Then
$$t = \frac{l_2}{\delta_2} \log_e(V-\rho_1) + \frac{m_1}{\delta_2} \log_e(V-\rho_2) + \frac{n_2}{\delta_2} \log_e(V-\rho_3) + \frac{C_2}{\delta_2}. \quad (101)$$

If
$$(V=V_1, t=t_1)$$
 is a point on the curve; that is, if $V=V$, when $t=t$, (102)

the time t at which the speed will attain some other value V is given by the equation,

$$t - t_{1} = \frac{l_{2}}{\delta_{2}} log_{e}(V - \rho_{1}) + \frac{m_{2}}{\delta_{2}} log_{e}(V - \rho_{2}) + \frac{n_{2}}{\delta_{2}} log_{e}(V - \rho_{3}) - \frac{l_{2}}{\delta_{3}} log_{e}(V, - \rho_{1}) - \frac{m_{2}}{\delta_{2}} log_{e}(V, - \rho_{2}) - \frac{n_{2}}{\delta_{2}} log_{e}(V, - \rho_{3}) .$$
(103)

Thus $t = t_1 + \frac{1}{\delta_2} \left[l_2 \log_e \frac{(V - \rho_1)}{(V_1 - \rho_2)} + m_2 \log_e \frac{(V - \rho_2)}{(V_1 - \rho_2)} + n_2 \log_e \frac{(V - \rho_3)}{(V_1 - \rho_3)} \right]. \quad (104)$

In common logarithms and expanded for convenience in computation,

$$t = t, + \frac{2.30}{\delta_2} \left[l_2 \log_{10}(V - \rho_1) + m_2 \log_{10}(V - \rho_2) + n_2 \log_{10}(V - \rho_3) - l_2 \log_{10}(V_1 - \rho_1) - m_2 \log_{10}(V_1 - \rho_2) - n_2 \log_{10}(V_1 - \rho_3) \right].$$
 (105)

Combining equations (101) and (105) and letting

$$C_2 = \frac{C_2'}{\delta_2} \quad , \tag{106}$$

$$C_z = \frac{C_z'}{\delta_z} = t, -\frac{l_2}{\delta_z} \log_e(V_r - \rho_r) - \frac{m_z}{\delta_z} \log_e(V_r - \rho_z) - \frac{n_2}{\delta_z} \log_e(V_r - \rho_z)$$
 (107)

or
$$C_2 = t_1 - \frac{2.30}{\delta_2} \left[l_2 log_{10}(V_1 - \rho_1) + m_2 log_{10}(V_1 - \rho_2) + n_2 log_{10}(V_1 - \rho_3) \right].$$
 (108)

Then, from equations (105) and (108),

$$t = C_2 + \frac{2.30}{\delta_2} \left[l_2 \log_{10}(V - \rho_1) + m_2 \log_{10}(V - \rho_2) + n_2 \log_{10}(V - \rho_3) \right] . \tag{109}$$

D - COASTING

In approaching stations, descending grades, slowing down for crossings, etc., it is generally advantageous from the standpoint of energy economy to shut off the electric power supply and allow the train to coast for a period before the brakes are applied. Under this condition, the motors do not supply any power to the train; in fact, they

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$$t = \frac{l_{1}}{\delta_{2}} \log_{e}(V-\rho_{0}) + \frac{m_{1}}{\delta_{2}} \log_{e}(V-\rho_{2}) + \frac{n_{2}}{\delta_{2}} \log_{e}(V-\rho_{3}) + \frac{c_{2}}{\delta_{2}}. \quad (101)$$

If $\{vev_1, vev_1\}$ is a point on the ourse that is, if V = V, when t = t,

the time t at which the eyeod will attain some other value V is given by the ecention.

$$t - t_1 = \frac{I_2}{\delta_2} log_0(V - \rho_1) + \frac{m_2}{\delta_2} log_0(V - \rho_2) + \frac{n_1}{\delta_2} log_0(V - \rho_3) + \frac{n_1}{\delta_2} log_0(V - \rho_3) - \frac{1}{\delta_2} log_0(V - \rho_3) .$$

$$= \frac{I_2}{\delta_2} log_0(V - \rho_1) - \frac{m_2}{\delta_2} log_0(V - \rho_2) - \frac{n_1}{\delta_2} log_0(V - \rho_3) .$$

$$= t_1 + \frac{I}{\delta_2} \left[I_2 log_0 \frac{(V - \rho_1)}{(V - \rho_1)} + m_2 log_0 \frac{(V - \rho_2)}{(V - \rho_2)} + n_2 log_0 \frac{(V - \rho_3)}{(V - \rho_3)} \right].$$

In common logarithms and expended for convenience in computation,

$$\dot{t} = \dot{t} + \frac{2.30}{6z} \left[l_z log_o(V-\rho) + m_z log_o(V-\rho_z) + n_z log_o(V-\rho_z) \right] - l_z log_o(V-\rho_z) - m_z log_o(V-\rho_z) - n_z log_o(V-\rho_z) \right].$$

Selected box (EOI) hom (IOI) emblemes settericos

$$C_{z} = \frac{C_{z}}{\delta_{z}} = t_{z} - \frac{I_{z}}{\delta_{z}} \log_{e}(W - \rho_{z}) - \frac{m_{z}}{\delta_{z}} \log_{e}(W - \rho_{z}) - \frac{n_{z}}{\delta_{z}} \log_{e}(W - \rho_{z})$$

$$C_2 = t_1 - \frac{2.30}{\delta_k} \left[l_1 log_{10} (V - \rho_1) + m_2 log_{10} (V_1 - \rho_2) + n_1 log_{10} (V_1 - \rho_3) \right].$$

(901) has (861) anoldance mort , ment

$$t = C_2 + \frac{2.30}{g_2} \left[i_x log_{10} (V - \rho_1) + m_z log_{10} (V - \rho_2) + n_z log_{10} (V - \rho_3) \right] \ .$$

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draw a certain small amount from the kinetic energy of the train in order to overcome the friction and windage losses in the motors and goars. However, this draught is usually negligible, so it may be assumed that the tractive effort input to the train is zero during coasting. Also, by the above definition, the briking effort is zero during coasting. Hence, with

(024)

and B = 0, (111)

the fundamental equation (1) reduces to

$$\frac{dV}{dt} = 0.01 \left[-C - G - \frac{50}{\sqrt{T}} - 0.03V - \frac{0.002X}{T} (1 + \frac{N-1}{10})V^2 \right]$$
 (112)

The value of G will be positive or negative according as the train is going up-hill or down-hill. The magnitude and algebraic sign of the value of G will determine whether the acceleration, $\frac{dV}{dt}$, is positive, negative or zero. Expressed algebraically:

$$\frac{dV}{dt} < 0 \quad if \quad G \ > -\left[C + \frac{50}{\sqrt{T}} + 0.03 \, V + \frac{0.002 \, X}{T} \left(I + \frac{N-I}{10}\right) \, V^2\right] \; ; \qquad (1/3)$$

$$\frac{dV}{dt} = 0 \quad \text{if} \quad G = -\left[C + \frac{50}{\sqrt{T}} + 0.03V + \frac{0.002X}{7} \left(1 + \frac{N-1}{10}\right)V^2\right]; \quad (114)$$

$$\frac{dV}{dt} > 0 \quad if \quad G < -\left[C + \frac{50}{\sqrt{T}} + 0.03V + \frac{0.002X}{T} \left(I + \frac{N-I}{10}\right)V^2\right]. \tag{115}$$

In equation (112) let

Then

$$\alpha_3 = 0.01 \left(C + G + \frac{50}{\sqrt{T}} \right) , \qquad (116)$$

$$\beta_3 = 0.0003$$
 , (117)

$$y_3 = \frac{0.00002 \, \text{X}}{T} \left(1 + \frac{N-1}{10} \right) \,. \tag{18}$$

$$\frac{dV}{dt} = -(\alpha_5 + \beta_3 V + \gamma_3 V^2) , \qquad (119)$$

$$-dt = \frac{dV}{\gamma_3 V^2 + \beta_3 V + \alpha_3} , \qquad (120)$$

$$-\frac{7}{3}dt = \frac{dV}{V^2 + \frac{1}{1}(3)} + \frac{\alpha_3}{7} + \frac{\alpha_3}{7}$$
 (121)

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al class ent to whene electic ent mil torone lines alutes a ment order to overgone the friction and vindage lesses in the motors and genra. However, this drought is morning negligible, so it may be assumed that the transity effort input to the brain is not guing consting. Also, by the chove definition, the belight offert is sero during occasing. Hence, with

of accomen (I) moltage latmenahmul end

$$\frac{dV}{dt} = 0.01 \left[-C - G - \frac{50}{\sqrt{17}} - 0.03V - \frac{0.002X}{T} (1 + \frac{N-1}{10})V^2 \right]$$
 (112)

ent as gailerooms svitegen to eviting of filly 0 to enter ent tooln is coing up-hill or down-bill. The wegatoone and algebraic sign of the value of G will determine whether the scooleration, is positive, negative or sero. Represent alcohestenius

$$\frac{dV}{dt} < 0 \quad H \quad G \quad > - \left[c + \frac{50}{\sqrt{7}} + 0.03 \, V + \frac{0.002 \times (1 + \frac{N+1}{10}) \, V}{10} \right] \, , \qquad (113)$$

$$\frac{dV}{dt} = 0 \quad if \quad G = -\left[c + \frac{s_0}{\sqrt{\tau}} + 0.03V + \frac{0.002X}{7}(1 + \frac{N-1}{10})V^{2}\right]; \quad (114)$$

$$\frac{dV}{dt} > 0 \quad \text{if} \quad 6 < -\left[c + \frac{30}{\sqrt{7}} + 0.03V + \frac{0.002 \times}{7}(l + \frac{N-1}{10})V^2\right]. \quad (115)$$

del (EII) neldamos ni

$$\alpha_3 = 0.01 \left(C + G + \frac{SO}{\sqrt{T}}\right), \qquad (116)$$

(811)
$$\frac{(1-N)^{1/2}}{7} = \frac{0.00000.0}{7} = \frac{1}{7}$$

$$\frac{dV}{dt} = -(a_3 + \beta_3 V + \beta_3 V^2) , \qquad (119)$$

$$-dt = \frac{dV}{2^4V^2 + \beta_3V + \alpha_9}$$

$$-dt = \frac{dV}{2^{3}V^{2} + \beta_{3}V + \alpha_{3}}$$

$$-\zeta dt = \frac{dV}{V^{2} + \frac{\beta_{3}}{2^{3}}V + \frac{\alpha_{3}}{2^{3}}}$$

$$(120)$$

 $-\gamma_3 dt = \frac{dV}{\left(V^2 + \frac{\beta_3}{\gamma_3}V + \frac{\beta_3^2}{4\gamma_3^2}\right) + \left(\frac{\alpha_3}{\gamma_3} - \frac{\beta_3^2}{4\gamma_3^2}\right)} , \qquad (122)$

$$= \frac{dV}{\left(V + \frac{\beta_3}{27_3}\right)^2 + \left(\frac{\alpha_3}{7_3} - \frac{\beta_3^2}{47_3^2}\right)} . \qquad (123)$$

(325)

$$d\left(V + \frac{\beta_3}{2f_1}\right) = dV \qquad (124)$$

there time is expressed in seconds and the inverse sine (sill2) in

Hence
$$-\frac{J_3 dt}{\left(V + \frac{B_3}{2J_3}\right)^2 + \left(\frac{\alpha_3}{J_3} - \frac{B_3^2}{4J_3^2}\right)} . \qquad (125)$$

There are three possible solutions of equation (125). The algebraic sign of $(\frac{\alpha_1}{7_3} - \frac{\beta_3^2}{4\beta_3^2})$ determines which is the proper solution. The values of β_3 and β_3 are positive as long as the train is moving. However, the value of α_3 is positive or negative according as $G > -(C + \frac{S_2}{\sqrt{7}})$ or $G < -(C + \frac{S_2}{\sqrt{7}})$. Nevertheless, α_3 , β_3 and β_3 are constants, fixed by track and operating conditions so it can readily be determined whether

$$\frac{\alpha_1}{7_3} - \frac{\beta_3^2}{47_3^2} > 0 , \qquad \frac{\alpha_3}{7_3} - \frac{\beta_3^2}{47_3^2} = 0 \quad or \quad \frac{\alpha_3}{7_3} - \frac{\beta_3^2}{47_3^2} < 0 .$$

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mitted in (127)

If
$$\frac{\alpha_3}{7_3} - \frac{\beta_3^2}{47_3^2} > 0$$
, (126)

the solution of
$$-\gamma_3 dt = \frac{d\left(V + \frac{\mathcal{O}_3}{2\gamma_3}\right)}{\left(V + \frac{\mathcal{O}_3}{2\gamma_3}\right)^2 + \left(\frac{\alpha_3}{\gamma_3} - \frac{\mathcal{O}_3^2}{4\gamma_3^2}\right)} \tag{125}$$

$$-7_3t = \frac{1}{\sqrt{\frac{\alpha'_3}{7_3} - \frac{\beta_3^2}{47_3^2}}} \sin^{-1}\left[\frac{V + \frac{\beta_3}{27_3}}{\sqrt{(V + \frac{\beta_3}{27_3})^2 + (\frac{\alpha_3}{7_3} - \frac{\beta_3^2}{47_3^2})}}\right] + C_3', \quad (127)$$

$$t = \frac{-2}{\sqrt{4\alpha_1\gamma_3 - \beta_3^2}} \sin\left[\frac{V + \frac{\beta_3}{275}}{\sqrt{V^2 + \frac{\beta_3}{73}V + \frac{\beta_3^2}{47_3^2} + \frac{\alpha_3}{73} - \frac{\beta_3^2}{47_3^2}}}\right] - \frac{C_3'}{7_3}$$
 (128)

$$Sin^{-1} is ok$$

$$Also -lan-1 oh if = \frac{-2}{\sqrt{4\alpha_{3}t_{3}-\beta_{3}^{2}}} sin^{2} \left[\frac{V + \frac{\beta_{3}}{2t_{3}}}{\sqrt{\frac{t}{7_{3}}(7_{3}V^{2} + \beta_{3}V + \alpha_{3})}} \right] - \frac{C_{3}^{2}}{7_{3}}$$

$$V + \frac{\beta_{3}}{2} + erm$$

$$(129)$$

$$= \frac{-2}{\sqrt{4\alpha_37_3 - \beta_3^2}} \sin^{-1} \left[\frac{27_3V + \beta_3}{\sqrt{47_3(7_3V^2 + \beta_3V + \alpha_3)}} \right] - \frac{C_3^2}{7_3} . \quad (130)$$

$$-1_{3}dt = \frac{V_{2}}{(V_{2} + \frac{1}{\sqrt{2}}) + (\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}})} + \frac{1}{\sqrt{2}}$$

$$(123) \qquad \frac{Vb}{(\sqrt{2} + \sqrt{2})^2 + (\sqrt{2} + \sqrt{2})} =$$

$$d(V+\frac{\beta_4}{2\beta_0})=dV . \qquad (124)$$

 $-Z_{dt} = \frac{d(V + \frac{Z_{3}}{Z_{3}})^{2} + (\frac{Z_{3}}{Z_{3}})^{2}}{d(V + \frac{Z_{3}}{Z_{3}})^{2} + (\frac{Z_{3}}{Z_{3}})^{2}} . \tag{125}$

There are three possible solutions of equation (125). The algebraic sign of (25) and (25) are constantly as (25) and (25) are constantly fixed by the are an operating as (25) and (25) are constantly for a position of the constantly are determined whether

$$\frac{\alpha_2}{\gamma_3} - \frac{\beta_2^*}{4\gamma_3^*} > 0 \quad , \quad \frac{\alpha_3}{\gamma_5} - \frac{\beta_2^*}{4\gamma_5^*} = 0 \quad or \quad \frac{\alpha_5}{\gamma_5} - \frac{\beta_2^*}{4\gamma_5^*} < 0 \; .$$

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$$\frac{\alpha_3}{\beta_3} - \frac{\beta_3^2}{4\beta_3^2} > 0$$
, (126)

the solution of
$$-t_0 dt = \frac{d(V + \frac{|Q_2|}{275})}{(V + \frac{|Q_2|}{275})^2 + (\frac{\alpha_2}{4} - \frac{|Q_2|}{245})}$$
 (125)

$$2e \qquad -3t = \frac{1}{\sqrt{\frac{6t_{3}}{4t_{3}} + \frac{6t_{3}}{4t_{3}}}} \sin \left(\frac{V + \frac{6t_{3}}{2t_{3}}}{(V + \frac{6t_{3}}{2t_{3}})^{2} + (\frac{6t_{3}}{t_{3}} - \frac{6t_{3}}{4t_{3}})} \right) + C_{3}^{2}, \quad (127)$$

$$t = \frac{-z}{\sqrt{4\pi s J_2 - J_3 z}} \sin \left[\frac{V + \frac{D_3}{D_3}}{V^2 + \frac{D_3}{D_3} + \frac{D_3}{A_3}} \right] - \frac{C_3^2}{J_3} (128)$$

$$= \frac{-2}{\sqrt{4\alpha_0 J_3 - \beta_0^2}} \sin \left[\frac{V + \frac{\beta_0}{2 J_3}}{\sqrt{\frac{1}{2} \left(J_3 \cdot V^2 + \beta_0 V + \alpha_0 \right)}} \right] - \frac{C_0^2}{J_0^2} \quad (12.9)$$

$$= \frac{-2}{\sqrt{4a_3b_5-p_3^2}} \sin \left[\frac{2f_5V+p_3}{\sqrt{4f_5(f_5V^2+p_3)V+a_3}} \right] - \frac{c_5^2}{f_5} . \quad (130)$$

where time is expressed in seconds and the inverse sine (\sin^{1}) in radians.

Since, in trigonometric tables, sines are compiled as
functions of angles expressed in degrees, it is more convenient to
modify the formula (150) so that the inverse sine can be derived
directly from the tables in degrees and decimal fractions of a
degree. This is accomplished by introducing the factor, 57.296,
the number of degrees in a radian. Equation (130) then becomes

$$t = \frac{-2}{57.3\sqrt{4\alpha_3\gamma_3 - \beta_3^2}} \sin^{-1} \left[\frac{2\gamma_3 V + \beta_3}{\sqrt{4\gamma_3(\gamma_5 V^2 + \beta_3 V + \alpha_3)}} \right] - \frac{C_3'}{\gamma_3} \qquad (/31)$$

$$= \frac{-1}{57.3\sqrt{4\alpha_3\gamma_3 - \beta_3^2}} \sin^{-1} \left[\frac{2\gamma_3 V + \beta_3}{\sqrt{4\gamma_3(\gamma_5 V^2 + \beta_3 V + \alpha_3)}} \right] - \frac{C_3'}{\gamma_3} \qquad (/32)$$

or $= \frac{-1}{28.6\sqrt{4\alpha_3} \gamma_3 - \beta_3^2} \sin^2 \left[\frac{2\gamma_3 V + \beta_3}{\sqrt{4\gamma_3} (\gamma_5 V^2 + \beta_3 V + \alpha_3)} \right] - \frac{C_3'}{\gamma_3}$ (132)

where time is expressed in seconds and the inverse sine in degrees.

If (t=t3, V=V3) be a point on the speed-time curve during the coasting period, that is, if

$$V = V_3$$
 when $t = t_3$; (133)

the time t, at which the train will attain any other speed V by coasting, is given by

$$t - t_3 = \frac{1}{28.6\sqrt{4\alpha_37_3 - \beta_3^2}} \sin^{-1}\left[\frac{27_3V_3 + \beta_3}{\sqrt{47_3(7_3V_3^2 + \beta_3V_3 + \alpha_3)}}\right] - \frac{1}{28.6\sqrt{4\alpha_37_3 - \beta_3^2}} \sin^{-1}\left[\frac{27_3V + \beta_3}{\sqrt{47_3(7_3V^2 + \beta_3V + \alpha_3)}}\right] (/34)$$

or
$$t = t_3 + \frac{1}{28.6\sqrt{4\alpha_3\beta_3 - \beta_3^2}} \left\{ si\vec{n} \left[\frac{2\beta_3V_3 + \beta_3}{\sqrt{47_3(\beta_3V_3^2 + \beta_3V_3 + \alpha_3)}} \right] - si\vec{n} \left[\frac{2\beta_3V + \beta_3}{\sqrt{4\gamma_3(\beta_3V_2 + \beta_3V + \alpha_3)}} \right] \right\}.$$
 (135)

Combining equations (132) and (155), and letting

$$C_3 = -\frac{C_3'}{7_3} , \qquad (/36)$$

(145)

$$C_{3} = t_{3} + \frac{1}{28.6\sqrt{4\alpha_{3}t_{3} - \beta_{3}^{2}}} sin^{2} \left[\frac{2t_{3}V_{3} + \beta_{3}}{\sqrt{4t_{3}(t_{3}V_{3}^{2} + \beta_{3}V_{3} + \alpha_{3})}} \right] , \quad (137)$$

and
$$t = C_3 - \frac{1}{28.6 \sqrt{4\alpha_3 f_3 + \beta_3^2}} \sin \left[\frac{27_3 V + \beta_3}{\sqrt{47_3 (f_3 V^2 + \beta_3 V + \alpha_3)}} \right]. \quad (138)$$

whore time is expressed in escends and the inverse sine (sin) in

Since, in trigonometric tables, since are conciled as functions of engine expressed in decrees, it is more convenient to modify the formula (130) so that the inverse sine one to derived directly from the tables in degrees and decimal fractions of a degree. This is accomplished by introducing the footer, ET.296, the marker of degrees in a radion. Equation (130) then became

$$t = \frac{-z}{57.3\sqrt{4\alpha_3 t_5 - \beta_3^2}} \sin \left[\frac{27.5V + \beta_3}{\sqrt{4\beta_3^2}(7.5V^2 + \beta_3 V + \alpha_3^2)} \right] - \frac{C_3^2}{\beta_3} \quad (131)$$

$$= \frac{-1}{26.5\sqrt{44}a_3b_5-35} \sin \left[\frac{2f_5V+\beta_5}{\sqrt{4f_5(f_5V^2+\beta_5V+\alpha_5)}}\right] - \frac{C_1^2}{f_5} \quad (132)$$

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the ecceting pariod, that is, if

$$V = V_{\rm S}$$
 when $v = v_{\rm S}$ (1993)

the time t, at which the train will attain may other speed Y by december, is given by

$$t - t_{s} = \frac{1}{286\sqrt{4037.703}} sin \left[\frac{27_{5}V_{5} + \rho_{3}}{\sqrt{47_{5}(7_{5}V_{5}^{2} + \rho_{3}V_{5} + \rho_{3})}} - \frac{1}{286\sqrt{4037_{5} + \rho_{3}^{2}}} sin \left[\frac{27_{5}V_{7} + \rho_{3}}{\sqrt{407_{5}(7_{5}V_{5}^{2} + \rho_{3}V_{5} + \rho_{3}^{2})}} \right] (13)$$

or
$$t = t_3 + \frac{1}{z_8.6 \sqrt{4\alpha_3 \delta_5 - \beta_3}} \left\{ \sin \left[\frac{z \gamma_5 V_3 + \beta_3}{\sqrt{4\gamma_5} \left(\gamma_5 V_5 + \beta_3 V_5 + \alpha_3 \right)} \right] - \sin \left[\frac{z \gamma_5 V + \beta_3}{\sqrt{4\gamma_5} \left(\gamma_5 V_5 + \beta_3 V_5 + \alpha_3 \right)} \right] \right\}.$$
 (13)

Combining equations (1881) and (1881, and leating

$$C_3 = -\frac{C_3^2}{7_3}$$
 (136)

$$C_{3} = t_{3} + \frac{1}{28.6\sqrt{200_{3}t_{3} - \beta_{3}^{2}}} \sin \left[\frac{27_{5}V_{3} + \beta_{3}}{\sqrt{27_{5}}(t_{5}V_{3}^{2} + \beta_{3}V_{4} + \alpha_{3})} \right] , \quad (137)$$

$$t = C_3 - \frac{1}{28.6 \sqrt{4} \alpha_3 \beta_3 + \beta_3^2} S(\tilde{n} \left[\frac{2 \beta_3 V + \beta_3}{\sqrt{4} \beta_3 (\beta_3 V^2 + \beta_3 V + \alpha_3)} \right]. \quad (138)$$

$$\frac{\alpha_3}{7_3} - \frac{\beta_3^2}{47_3^2} = 0 \quad , \tag{139}$$

equation (125) becomes
$$\gamma_s dt = \frac{-d\left(V + \frac{\beta_s}{2T_s}\right)}{\left(V + \frac{\beta_s}{2T_s}\right)^2} . \tag{140}$$

The solution of this is
$$7_3 t = \frac{1}{V + \frac{\beta_3}{27_3}} + C_3'' \qquad (141)$$

$$t = \frac{2}{2\gamma_3 V + \beta_3} + \frac{C_3''}{\gamma_3} . \tag{142}$$

If
$$V = V_3$$
 when $t = t_3$, (133)

the time t, at which the train will reach any other speed V by coasting,

is given by
$$t - t_3 = \frac{2}{2 t_3 V + \beta_3} - \frac{2}{2 t_3 V_3 + \beta_3}$$
 (143)

$$t = t_3 + \frac{2}{27_3 V + \beta_3} - \frac{2}{27_3 V_3 + \beta_3} . \qquad (144)$$

Combining equations (142) and (144), and letting

$$C_3 = \frac{C_3''}{\gamma_3} , \qquad (14.5)$$

(151)

(152)

(153)

$$C_3 = t_3 - \frac{2}{27_3 V_3 + \beta_3}$$
 (146)

$$t = C_3 + \frac{2}{27_3V_3 + \beta_3} \cdot (148)$$
Then
$$t = C_3 + \frac{2}{27_3V + \beta_3} \cdot (147)$$

possible, is given by

If
$$\frac{\alpha_3}{7_3} - \frac{\beta_3^2}{47_3^2} < 0$$
, (148)

= 4 + 2:00 (log Viens-121+0+24V) -log Viens-13+0+274V (log

$$\frac{\alpha_3}{7_3} - \frac{\beta_3^2}{47_3^2} = -\left| \frac{\alpha_3}{7_3} - \frac{\beta_3^2}{47_3^2} \right| \tag{149}$$

where the parallel vertical lines signify that the absolute, or numerical, value or the quantity, that they enclose, is taken with

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$$\frac{\alpha_{s}}{\gamma_{s}^{2}} - \frac{\beta_{s}^{2}}{4\gamma_{s}^{2}} = 0 \quad , \tag{139}$$

equation (125) becomes
$$f_{i}dt = \frac{-d(V + \frac{\beta_{i}}{2\beta_{i}})}{(V + \frac{\beta_{i}}{2\beta_{i}})^{2}}$$
. (140)

The solution of this is $t_s t = \frac{1}{(V + \frac{R_s}{2K}} + C_s^{\prime\prime\prime} \qquad (141)$

$$t = \frac{2}{27V + \beta_3} + \frac{C_3^n}{\beta_3} \,, \tag{142}$$

If
$$V = V_0$$
 when $b - b_0$.

the time t, at which the train will reach say other speed V by consting,

10 given by
$$t-t_5 = \frac{2}{25V+\beta_3} - \frac{2}{25V_3+\beta_3}$$
 (143)

$$t = t_3 + \frac{2}{2J_3V + \beta_3} - \frac{2}{2J_3V_3 + \beta_3} . \quad (144)$$

Combining equations (142) and (143), and lebiling

$$C_3 = \frac{C_3^*}{T_2}$$
, (145)

$$C_3 = t_3 - \frac{2}{2t_3 V_4 \mu_3}$$
 (146)

Then
$$t = C_0 + \frac{2}{2KV + \beta_0}. \qquad (147)$$

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$$\frac{d_{2}}{7_{2}} - \frac{\partial_{3}^{2}}{47_{3}^{2}} < 0 , \qquad (148)$$

$$\frac{\alpha_3}{7_3} - \frac{\beta_3^2}{47_5^2} = -\frac{\alpha_3}{7_3} - \frac{\beta_3^2}{47_5^2} \tag{14.9}$$

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the positive algebraic sign.

Substituting the relation (149) into equation (125), gives

$$-\frac{7}{3}dt = \frac{d(V + \frac{\beta_3}{27_3})}{(V + \frac{\beta_3}{27_3})^2 - \left|\frac{\alpha_3}{7_3} - \frac{\beta_3^3}{47_3^3}\right|}$$
(150)

or

$$\gamma_{3} dt = \frac{d(V + \frac{\beta_{3}}{2\gamma_{3}})}{\left|\frac{\alpha_{3}}{\gamma_{3}} - \frac{\beta_{3}^{2}}{4\gamma_{3}^{2}}\right| - \left(V + \frac{\beta_{3}}{2\gamma_{3}}\right)^{2}} . \tag{151}$$

The solution of this is

$$7_{3}t = \frac{1}{2\sqrt{\left|\frac{\alpha_{3}}{7_{3}} - \frac{\beta_{3}^{2}}{4f_{3}^{2}}\right|}} \log_{e} \left[\frac{\sqrt{\left|\frac{\alpha_{3}}{7_{3}} - \frac{\beta_{3}^{2}}{4f_{3}^{2}}\right|} + \left(V + \frac{\beta_{3}}{2f_{3}}\right)}{\sqrt{\left|\frac{\alpha_{3}}{7_{3}} - \frac{\beta_{3}^{2}}{4f_{3}^{2}}\right|} - \left(V + \frac{\beta_{3}}{2f_{3}}\right)} \right] + C_{3}^{"'} \cdot (152)$$

$$t = \frac{1}{\sqrt{|4\alpha_3 \gamma_3 - \beta_3^2|}} \log_e \left[\frac{\sqrt{|4\alpha_3 \gamma_3 - \beta_3^2|} + \beta_3 + 2\gamma_5 V}{\sqrt{|4\alpha_3 \gamma_3 - \beta_3^2|} - \beta_3 - 2\gamma_3 V} \right] + \frac{C_3'''}{\gamma_3} . \quad (153)$$

In common logarithme, in order to facilitate computation,

$$t = \frac{2.30}{\sqrt{|4\alpha_3J_3 - \beta_3^2|}} \log_{10} \left[\frac{\sqrt{|4\alpha_3J_3 - \beta_3^2|} + \beta_3 + 2J_3V}{\sqrt{|4\alpha_3J_3 - \beta_3^2|} - \beta_3 - 2J_3V} \right] + \frac{C_3'''}{J_3} . \tag{154}$$
If $V = V_3$ when $t = t_3$,

the time t, at which the train will attain any other speed V by

coasting, is given by

$$t - t_{3} = \frac{2.30}{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|}} \left\{ log_{10} \left[\frac{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|} + \beta_{3} + 2\gamma_{3}V}{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|} - \beta_{3} - 2\gamma_{3}V} \right] - log_{10} \left[\frac{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|} + \beta_{3} + 2\gamma_{3}V_{3}}{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|} + \beta_{3} - 2\gamma_{3}V_{3}} \right] \right\}, (155)$$

so that

$$t = t_{2} + \frac{2.30}{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|}} \left\{ log_{10} \left[\frac{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|} + \beta_{3} + 27_{3}V}{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|} - \beta_{3} - 27_{3}V} \right] - log_{10} \left[\frac{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|} + \beta_{3} + 27_{3}V_{3}}{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|} - \beta_{3} - 27_{3}V_{3}} \right] \right\}. (156)$$

Combining equations (154) and (156), and letting

$$C_3 = \frac{C_3'''}{7_3} \quad , \tag{166}$$

$$C_{3} = t_{3} - \frac{2.30}{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|}} \log_{10} \left[\frac{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|} + \beta_{3} + 27_{3}V_{3}}{\sqrt{|4\alpha_{3}7_{3} - \beta_{3}^{2}|} - \beta_{3} - 27_{3}V_{3}} \right], \qquad (158)$$

$$t = C_3 + \frac{2.30}{\sqrt{|4\alpha_3\beta_3 - \beta_3^2|}} log_{10} \left[\frac{\sqrt{|4\alpha_3\beta_3 - \beta_3^2|} + \beta_3 + 2\beta_3 V}{\sqrt{|4\alpha_3\beta_3 - \beta_3^2|} - \beta_3 - 2\beta_3 V} \right].$$
 (155)

the positive eigelmain eign.

substituting the relation (149) into equation (135), gives

$$-idt = \frac{d(V + \frac{2i}{2i})}{(V + \frac{2i}{2i})^2 - \left|\frac{\alpha_i}{\alpha_i} - \frac{\alpha_i^2}{\alpha_i^2}\right|}$$
(15)

The solution of this is

$$Z_{z}^{z} = \frac{1}{2\sqrt{\frac{a_{z}^{z}}{A_{z}^{z}}}} \log \left[\frac{\sqrt{\frac{a_{z}^{z}}{A_{z}^{z}}} + (V + \frac{a_{z}^{z}}{A_{z}^{z}})}{\sqrt{\frac{a_{z}^{z}}{A_{z}^{z}}} - \frac{a_{z}^{z}}{A_{z}^{z}}} \right] + C_{y}^{z}^{z} \cdot (15)$$

The term of
$$t = \frac{1}{\sqrt{4\alpha_3\beta_3 - \beta_3^2}} \log_e \left[\frac{\sqrt{4\alpha_3\beta_3 - \beta_3^2} + \beta_3 + 2\beta_3 V}{\sqrt{4\alpha_3\beta_3 - \beta_3^2} + \beta_3 - 2\beta_3 V} \right] + \frac{C_3'''}{\beta_3}$$
. (15)

In common logarithms, in order to facilitate computation,

$$t = \frac{2.30}{\sqrt{|4\alpha_3 Y_3 - \beta_3^2|}} \log_0 \left[\frac{\sqrt{|4\alpha_3 Y_3 - \beta_3^2|} + \alpha_3 + 2\gamma_3 V}{\sqrt{|4\alpha_3 Y_3 - \beta_3^2|} - \beta_3 - 2\gamma_3 V} \right] + \frac{C_5^2}{\gamma_3} .$$
 (12)

$$M = V_{\rm S} \text{ without } b = b_{\rm S} \text{ *} \tag{203}$$

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doneting, is given by

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$$t-t_{s}=\frac{2.30}{\sqrt{|4\alpha_{s}\beta_{s}-\beta_{s}^{2}|}+\beta_{s}+2\beta_{s}V}}-l_{0}g_{s}\left[\frac{\sqrt{|4\alpha_{s}\beta_{s}-\beta_{s}^{2}|}+\beta_{s}+2\beta_{s}V}}{\sqrt{|4\alpha_{s}\beta_{s}-\beta_{s}^{2}|}+\beta_{s}+2\beta_{s}V}}\right],\;(1)$$

Completel one (164) and (166), and letting

$$C_9 = \frac{C_5^m}{75} \quad ,$$

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$$t = C_3 + \frac{2.30}{\sqrt{4\alpha_3 t_3 - \beta_3^2}} \log_{10} \sqrt{\frac{14\alpha_3 t_3 - \beta_3^2 + \beta_3 + 2\xi V}{\sqrt{4\alpha_3 t_3 - \beta_3^2} - \beta_3 - 2\xi V}}.$$
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E - BRAKING

When the brakes are applied to a train, the power supply is usually shut off as in coasting, so the input tractive effort, F in equation (1) is zero during the braking period. Hence, equation (1) becomes

$$\frac{dV}{dt} = 0.01 \left[-B - C - G - \frac{50}{\sqrt{T}} - 0.03V - \frac{0.002X}{T} (1 + \frac{N-1}{10})V^2 \right]. \quad (160)$$

Brakes are most commonly applied for the purpose of retarding the motion of a train. However, occasionally, as during the descent
of grades, the brakes are partially applied and the speed allowed to
increase though not to such an extent as it would if the brakes were
not applied. Therefore, in order to be general, the formulae for speedtime relations during braking must provide for positive, zero and negative acceleration. Expressed algebraically:

$$\frac{dV}{dt} < 0 \quad \text{if} \quad G > -\left[8 + C + \frac{50}{\sqrt{T}} + 0.03V + \frac{0.002 \, \text{X}}{T} \left(1 + \frac{N-1}{10}\right) V^2\right]; \tag{161}$$

$$\frac{dV}{dt} = 0 \quad \text{if} \quad G = -\left[B + C + \frac{50}{\sqrt{T}} + 0.03V + \frac{0.002X}{T} \left(I + \frac{N-I}{10}\right)V^2\right]; \quad (162)$$

$$\frac{dV}{dt} > 0 \quad \text{if} \quad G < -\left[B + C + \frac{50}{\sqrt{T}} + 0.03V + \frac{0.002X}{T} \left(1 + \frac{N-1}{10}\right)V^2\right]. \tag{163}$$

The acceleration is

$$A = \frac{dV}{dt} = 0.01 \left[-B - C - G - \frac{50}{\sqrt{T}} - 0.03V - \frac{0.002X}{T} (1 + \frac{N-1}{10})V^2 \right] \cdot (164)$$

Let
$$\alpha_A = 0.01 \left(B + C + G + \frac{50}{\sqrt{T}} \right)$$
, (165)

$$\beta_4 = 0.0003$$
 , (166)

$$\gamma_{4} = \frac{0.00002X}{T} \left(1 + \frac{N-1}{10} \right) . \tag{167}$$

Thon

$$\frac{dV}{dt} = -(\alpha_4 + \beta_4 V + \gamma_4 V^2) \qquad (168)$$

then the browns are applied to a train, the power supply is some in the country about of a the country about the series the training the training partod. Hondo, countries (1)

$$\frac{dV}{dt} = 0.0i \left[-8 - c - 6 - \frac{80}{\sqrt{7}} - 0.03V - \frac{0.002X}{T} (1 + \frac{N-1}{10})V^2 \right]. \quad (160)$$

present are mosted of a busin. Mosever, consideredly, as during the descent of grades, the mosted of a busine are partially explied and the epoch allowed to increase therein not to another as it would if the breaker were not applied. Therefore, in order to be general, the formales for apost-time relations during breating wast provide for positive, note and magnify the secoloration. Expressed also breakerships.

$$\frac{dV}{dt} < 0 \quad \text{if} \quad G > -\left[B + C + \frac{50}{\sqrt{7}} + 0.03V + \frac{0.002 \, X}{7} (1 + \frac{N-1}{10}) V^2 \right]; \quad (161)$$

$$\frac{dV}{dt} = 0 \quad \text{if} \quad G = -\left[B + C + \frac{SO}{\sqrt{7}} + 0.03V + \frac{0.002X}{T} (1 + \frac{N - 1}{10})V^2\right]; \quad (162)$$

$$\frac{dV}{dt} > 0 \quad H \quad G < -\left[B + C + \frac{50}{\sqrt{T}} + 0.03V + \frac{0.002X}{7} \left(l + \frac{N-l}{10} \right) V^2 \right]. \quad (163)$$

The ecocloration is

$$A = \frac{dV}{dt} = 0.01 \left[-8 + c - c - \frac{50}{\sqrt{T}} - 0.03V - \frac{0.002X}{T} (1 + \frac{N-1}{10})V^{2} \right]. \quad (164)$$

Let
$$\alpha_{e} = 0.01 \left(B + C + G + \frac{50}{\sqrt{\pi}}\right), \qquad (165)$$

$$\beta_4 = 0.0003$$
 , (166)

$$(181) \frac{(1-N+1)^{\frac{N-1}{2}}}{T} = \frac{1}{2}$$

Marien.

$$\frac{dV}{dt} = -(\alpha_0 + \beta_0 V + 2 V^2) . (166)$$

A comparison of equations (116) to (119) and (165) to (168) shows that the speed-time relations during the coasting period are different from those in the braking period only in that $B \neq 0$ in the latter; that is

$$\alpha_4 = 0.01(B + C + G + \frac{50}{\sqrt{T}})$$
 (165)

(124)

while
$$\alpha_s = 0.01(C + G + \frac{50}{\sqrt{T}}) . \tag{116}$$

Hence, since and are both constants, the solutions for the two periods will be similar in form and the resulting formulae for the braking period can be written directly.

Thus, if (t-t₄, V=V₄) be a point on the speed-time curve in the braking period, that is, if

$$V = V_4$$
 when $t = t_4$, (169)

the time t, at which the speed will attain some other value V under constant application of the brakes, is given by the following formulae:

Case I

$$\frac{\alpha_4}{\gamma_4} - \frac{\beta_4^2}{4\gamma_4^2} > 0 , \qquad (170)$$

$$t = t_4 + \frac{1}{28.6\sqrt{4\alpha_4 l_4 - \beta_4^2}} \left\{ \sin^{-1} \left[\frac{2\gamma_4 V_4 + \beta_4}{\sqrt{4\gamma_4 (\gamma_4 V_4^2 + \beta_4 V_4 + \alpha_4)}} \right] - \sin^{-1} \left[\frac{2\gamma_4 V + \beta_4}{\sqrt{4\gamma_4 (\gamma_4 V_4^2 + \beta_4 V_4 + \alpha_4)}} \right] \right\}. (171)$$

Letting
$$C_4 = t_4 + \frac{1}{28.6\sqrt{4\alpha_4 t_4 - \beta_4^2}} \sin^2 \left[\frac{2t_4 V_4 + \beta_4}{\sqrt{4t_4 (t_4^2 V_4^2 + \beta_4 V_4 + \alpha_4)}} \right]$$
, (172)

$$t = C_4 - \frac{1}{28.6\sqrt{4\alpha_4 J_4^3 - \beta_4^2}} \sin^{-1} \left[\frac{2J_4V + \beta_4}{\sqrt{4J_4(J_4V^2 + \beta_4V + \alpha_4)}} \right]. \tag{173}$$

In these formulae, time is expressed in seconds, speed in miles per hour, and inverse sines in degrees.

A comperison of equations (116) to (119) and (168) to (168) to enough the special case where the the theoretical case the tree there is the braining particle only in that is f = 0 in the latter; that is f = 0 or f = 0 or f = 0.

while we have the second
$$\alpha_s = 0.01(B+C+G+\frac{SO}{\sqrt{T}})$$
 and $\alpha_s = 0.01(C+G+\frac{SO}{\sqrt{T}})$.

Homes, since and are both constants, the solutions for the two periods will be similar in form and the resulting forquise for the breaking period can be written directly.

rows, if (tot, , VeV,) to a point on the specific curve in the broking period, that is, if

 $t = t_a + \frac{\alpha_a}{zse\sqrt{4\alpha_at_a} - \beta_a^2} \left\{ sin' \left[\frac{2t_aV_a + \beta_a}{\sqrt{4\pi_a'(t_aV_a' + \beta_aV_a' + \beta_a)}} \right] - sin' \left[\frac{2t_aV_a + \beta_a}{\sqrt{4\pi_a'(t_aV_a' + \beta_aV_a' + \alpha_a)}} \right] \right\}. (i)$ Letter $C_a = t_a + \frac{1}{zs.6\sqrt{4\alpha_at_a} - \beta_a^2} sin' \left[\frac{2t_aV_a + \beta_a}{\sqrt{4\pi_a'(t_aV_a' + \beta_aV_a' + \alpha_a)}} \right]. (i)$ $t = C_a - \frac{1}{zs.6\sqrt{4\alpha_aC_a - \beta_a^2}} sin' \left[\frac{2t_bV_a + \beta_a}{\sqrt{4\pi_a'(t_aV_a' + \beta_aV_a' + \alpha_a)}} \right]. (i)$

In those formulas, time is expressed in seconds, spent in miles par how, and inverse since in degrees.

If
$$\frac{\alpha_4}{\gamma_4} - \frac{\beta_4^2}{4\gamma_2^2} = 0$$
, (174)

$$t = t_4 + \frac{2}{2\frac{7}{4}V + \beta_4} - \frac{2}{2\frac{7}{4}V_4 + \beta_4}. \tag{175}$$

Letting
$$C_4 = t_4 - \frac{2}{2 t_4 V_4 + B_4}$$
, (176)

$$t = C_4 + \frac{2}{274V + \beta_4} . \tag{177}$$

Case III

If
$$\frac{\alpha_4}{\gamma_4} - \frac{\beta_4^2}{4\eta_4^2} < 0$$
, (178)

so that
$$\frac{\alpha_{4}}{7_{4}} - \frac{\beta_{4}^{2}}{47_{4}^{2}} = -\left|\frac{\alpha_{4}}{7_{4}} - \frac{\beta_{4}^{2}}{47_{4}^{2}}\right|, \qquad (179)$$

$$t = t_{4} + \frac{2.30}{\sqrt{|4\alpha_{4}7_{4} - \beta_{4}^{2}|}} \left\{ log_{10} \left[\frac{\sqrt{|4\alpha_{4}7_{4} - \beta_{4}^{2}|} + \beta_{4} + 27_{4}V}{\sqrt{|4\alpha_{4}7_{4} - \beta_{4}^{2}|} - \beta_{4} - 27_{4}V} \right] - log_{10} \left[\frac{\sqrt{|4\alpha_{4}7_{4} - \beta_{4}^{2}|} + \beta_{4} + 27_{4}V_{4}}{\sqrt{|4\alpha_{4}7_{4} - \beta_{4}^{2}|} - \beta_{4} - 27_{4}V_{4}} \right] \right\}.$$

Letting
$$C_4 = t_4 - \frac{2.30}{\sqrt{|4\alpha_4 T_4 - \beta_4^2|}} \log_{10} \left[\frac{\sqrt{|4\alpha_4 T_4 - \beta_4^2|} + \beta_4 + 2T_4 V_4}{\sqrt{|4\alpha_4 T_4 - \beta_4^2|} - \beta_4 - 2T_4 V_4} \right],$$
 (181)

$$t = C_4 + \frac{2.30}{\sqrt{|4\alpha_4}7_4 - \beta_4^2|} log_{10} \left[\frac{\sqrt{|4\alpha_4}7_4 - \beta_4^2|} {\sqrt{|4\alpha_4}7_4 - \beta_4^2|} + \beta_4 + 27_4 V \right] . \quad (182)$$

(135)

(125)

(100)

Special Case, V = 0 S V = 0 S V = 0 Special Case, V = 0 S

In stopping a train by the application of friction brakes, if the speed is V_4 at the instant t_4 in the braking period, the time t, at which the train will stop, is given by letting V=0 in equation (171). Then

$$t = t_4 + \frac{1}{28.6\sqrt{4\alpha_47_4 - \beta_4}} \left\{ \sin^{-1} \left[\frac{27_4V_4 + \beta_4}{\sqrt{47_4(7_4V_4^2 + \beta_4V + \alpha_4)}} \right] - \sin^{-1} \left[\frac{\beta_4}{\sqrt{4\alpha_47_4}} \right] \right\}$$
 (183)

$$\frac{\alpha_4}{\gamma_4} - \frac{\alpha_5}{\alpha_1^2} = 0 \quad ,$$

$$t = t_{+} + \frac{2}{2\chi V + \beta_{+}} - \frac{2}{2\chi V + \beta_{+}}, \quad (175)$$

(174)

(176)

(178)

(179)

(181)

(581)

Lebting

$$t = C_a + \frac{2}{276V + \rho_a} \quad (177)$$

$$\alpha_{e}$$
 β_{e}^{*} = $- |\alpha_{e} - \beta_{e}^{*}|$

 $\frac{a_a}{7a} - \frac{\beta_a^2}{47} < 0 \quad ,$

$$\frac{\alpha_{c}}{\gamma_{c}} - \frac{\beta_{c}^{2}}{4\gamma_{c}^{2}} = - \frac{\alpha_{c}}{\gamma_{c}} - \frac{\beta_{c}^{2}}{4\gamma_{c}^{2}}$$
,

$$t = t_{x} + \frac{2.30}{\sqrt{20245 - \beta_{x}^{2}}} \left\{ log_{0} \left[\frac{\sqrt{2025 - \beta_{x}^{2}} + \beta_{x} + 274}{\sqrt{2024 - \beta_{x}^{2}} - \beta_{x} - \beta_{x}^{2}} \right] - log_{0} \left[\frac{\sqrt{2024 - \beta_{x}^{2}} + \beta_{x} + 2746}{\sqrt{2024 - \beta_{x}^{2}} - \beta_{x} - 2746} \right] \right\}$$

Letters
$$C_a = t_a - \frac{2.30}{\sqrt{4\alpha_a x_a - \rho_a^2}} \log_{\infty} \left[\frac{\sqrt{4\alpha_a x_a - \rho_a^2} + \rho_a + 2\lambda_a N_a}{\sqrt{4\alpha_a x_a - \rho_a^2} + \rho_a + 2\lambda_a N_a} \right],$$

$$t = C_0 + \frac{2.30}{\sqrt{4\alpha_0 T_0 - \beta_0^2}} log_0 \left[\frac{\sqrt{4\alpha_0 T_0 - \beta_0^2} + \beta_0 + 2T_0 V}{\sqrt{4\alpha_0 T_0 - \beta_0^2} - \beta_0 - 2T_0 V} \right].$$

Special Case, V = 0

mestered meltoir? to meltesligge and ad misses springed and if the speed is Va et the instant ta in the braising period, the time t, at which the train will atop, is given by letting T = O in ment .(IVI) moldaune

$$t = t_a + \frac{1}{28.6\sqrt{4\alpha_a T_b - \beta_a}} \left\{ \sin \left[\frac{2f_a V_b + \beta_a}{\sqrt{4\alpha_b}(C_b V_a^2 + \beta_b V + \alpha_a)} \right] - \sin \left[\frac{\beta_a}{\sqrt{4\alpha_a T_b}} \right] \right\} \quad (183)$$

F - SUMMARY OF PRINCIPAL FORMULAR

In conclusion, the principal formulae are grouped together in order to facilitate reference to them.

The acceleration formula , or fundamental differential equation of train speed is

$$\frac{dV}{dt} = 0.01 \left[F - B - C - G - \frac{50}{\sqrt{T}} - 0.03V - \frac{0.002X}{T} (I + \frac{N-I}{10})V^2 \right]. \tag{1}$$

The formula for the tractive effort input to the train, when rated voltage is applied to the motor terminals, is

$$F = \frac{h_1}{V - h_2} \tag{22}$$

(148)

(170)

(374)

For the starting period, that is, with constant tractive

effort input to the train,

$$t = t_{o} + \frac{2.30}{\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}}} \begin{bmatrix} log_{io}(\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}} + \beta_{i} + 27_{i}V) - log_{io}(\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}} - \beta_{i} - 27_{i}V) \\ -log_{io}(\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}} + \beta_{i} + 27_{i}V_{o}) + log_{io}(\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}} - \beta_{i} - 27_{i}V_{o}) \end{bmatrix} .$$
(39)

If $V_0 = 0$ and $t_0 = 0$

11

$$t_{s} = \frac{2.30}{\sqrt{4\alpha,7,+\beta,^{2}}} \left[log_{10}(2\alpha_{1}-\beta,V+V\sqrt{4\alpha,7,+\beta,^{2}}) - log_{10}(2\alpha,-\beta,V-V\sqrt{4\alpha,7,+\beta,^{2}}) \right] . \quad (47)$$

During acceleration with rated voltage applied to the motors,

$$t = t_1 + \frac{1}{\delta_2} \left[l_2 \log_e \frac{V - \rho_1}{V_1 - \rho_1} + m_2 \log_e \frac{V - \rho_2}{V_1 - \rho_2} + n_2 \log_e \frac{V - \rho_3}{V_1 - \rho_3} \right] . \quad (104)$$

During consting,

$$t = t_{3} + \frac{1}{28.6\sqrt{4\alpha_{3}\gamma_{3} - \beta_{3}^{2}}} \left\{ \sin^{-1} \left[\frac{2\gamma_{3}V_{3} + \beta_{3}}{\sqrt{4\gamma_{3}(\gamma_{3}V_{3}^{2} + \beta_{3}V_{3} + \alpha_{3})}} \right] - \sin^{-1} \left[\frac{2\gamma_{3}V + \beta_{3}}{\sqrt{4\gamma_{3}(\gamma_{3}V^{2} + \beta_{3}V + \alpha_{3})}} \right] \right\}$$
 (135)

provided
$$\frac{\alpha_3}{\gamma_3} - \frac{\beta_3^2}{4\gamma_3^2} > 0 \quad ; \tag{126}$$

$$t = t_3 + \frac{2}{27_3 V + \beta_3} - \frac{2}{27_3 V_3 + \beta_3}$$
 (144)

$$\frac{\alpha_3}{\gamma_3} - \frac{\beta_3^2}{4\gamma_3} = 0 \quad ; \tag{139}$$

F SUBMER OF PRINCIPAL RESIDERS

In apparation, the principal formulae are grouped togother in order to facilitate reference to them.

the ecceleration formula: , or freshmental Afficeanticl al beens alers to meldance

$$\frac{dV}{dt} = 0.01 \left[F - B - C - G - \frac{SO}{\sqrt{T}} - 0.03V - \frac{O.002X}{T} \left(I + \frac{N-I}{IO} \right) V^2 \right].$$

the formula for the tractive effort input to the train, show el . sienteret rotte est co felige al egation beter

$$F = \frac{h}{V - h_2}.$$

For the starting period, that is, with constant tractive

effort input to the brein.

$$t = t_o + \frac{2.30}{\sqrt{4a_s \chi_1 + \beta_2^2}} \left[\frac{\log_{10}(\sqrt{4a_s \chi_1 + \beta_2^2} + \beta_1 + 2\chi V) - \log_{10}(\sqrt{4a_s \chi_1 + \beta_2^2} - \beta_1 - 2\chi V)}{-\log_{10}(\sqrt{4a_s \chi_1 + \beta_2^2} + \beta_1 + 2\chi V_o) + \log_{10}(\sqrt{4a_s \chi_1 + \beta_2^2} - \beta_1 - 2\chi V_o)} \right].$$

O = at hen O = AV 21

$$t_{s} = \frac{2.30}{\sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}}} \left[log_{io}(2\alpha_{i} - \beta_{i}V + V - \sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}}) - log_{io}(2\alpha_{i} - \beta_{i}V - V - \sqrt{4\alpha_{i}7_{i} + \beta_{i}^{2}}) \right] . \tag{4}$$

parties according the rate voltage applied to the motore,

$$t = t_1 + \frac{1}{d_2} \left[l_2 l_0 g_0 \frac{V - \rho_1}{V_1 - \rho_1} + m_1 l_0 g_0 \frac{V - \rho_2}{V_1 - \rho_2} + m_2 l_0 g_0 \frac{V - \rho_3}{V_1 - \rho_3} \right] . \quad (104)$$

Juring coseting.

$$t = t_s + \frac{1}{28.6 \sqrt{4\alpha_s \beta_s - \beta_s^2}} \left\{ \sin \left[\frac{2\beta_s V_s + \beta_s}{\sqrt{4\beta_s (\beta_s V_s + \beta_s V_s + \alpha_s)}} \right] - \sin \left[\frac{2\beta_s V_s + \beta_s}{\sqrt{4\beta_s (\beta_s V_s + \beta_s V_s + \alpha_s)}} \right] \right\}$$
or evidence.

$$\frac{\alpha_3}{\gamma_3} - \frac{\beta_3^2}{4\gamma_3^2} > 0 \quad ;$$

$$t = t_3 + \frac{2}{2t_3V + \beta_3} - \frac{2}{2t_3V_3 + \beta_3}$$

[12

(13E

$$\frac{d_3}{f_3} - \frac{g_3^2}{4f_3} = 0 \quad ;$$

and
$$t_{3} = t_{3} + \frac{2.30}{\sqrt{|4\alpha_{3}f_{3} - \beta_{3}^{2}|}} \begin{bmatrix} \log_{10}(\sqrt{|4\alpha_{3}f_{3} - \beta_{3}^{2}|} + \beta_{3} + 2f_{3}V) - \log_{10}(\sqrt{|4\alpha_{3}f_{3} - \beta_{3}^{2}|} - \beta_{3} - 2f_{3}V) \\ -\log_{10}(\sqrt{|4\alpha_{3}f_{3} - \beta_{3}^{2}|} + \beta_{3} + 2f_{3}V_{3}) + \log_{10}\sqrt{|4\alpha_{3}f_{3} - \beta_{3}^{2}|} - \beta_{3} - 2f_{3}V_{3}) \end{bmatrix}$$
(156)

$$\frac{\alpha_3}{\gamma_3} - \frac{\beta_3^2}{4\gamma_3^2} < 0 \quad . \tag{148}$$

During braking.

$$t = t_4 + \frac{1}{28.6\sqrt{4\alpha_4\eta_4 - \beta_4^2}} \left\{ si\vec{n} \left[\frac{2\gamma_4 V_4 + \beta_4}{\sqrt{4\gamma_4(\gamma_4 V_4^2 + \beta_4 V_4 + \alpha_4)}} \right] - si\vec{n} \left[\frac{2\gamma_4 V + \beta_4}{\sqrt{4\gamma_4(\gamma_4 V_4^2 + \beta_4 V_4 + \alpha_4)}} \right] \right\}$$
 (171)

provided
$$\frac{\alpha_4}{\gamma_4} - \frac{\beta_4^2}{4\gamma_2} > 0 \quad ; \tag{170}$$

$$t = t_4 + \frac{2}{27_4 V + \beta_4} - \frac{2}{27_4 V + \beta_4} \tag{175}$$

$$\frac{\alpha_{4}}{\gamma_{4}} - \frac{\beta_{4}^{2}}{4\gamma_{4}^{2}} = 0 \quad ; \tag{174}$$

and
$$t = t_4 + \frac{2.30}{\sqrt{|4\alpha_47_4 - \beta_4^2|}} \begin{bmatrix} log_{10}(\sqrt{|4\alpha_47_4 - \beta_4^2|} + \beta_4 + 27_4V) - log_{10}(\sqrt{|4\alpha_47_4 - \beta_4^2|} - \beta_4 - 27_4V) \\ -log_{10}(\sqrt{|4\alpha_47_4 - \beta_4^2|} + \beta_4 + 27_4V_4) + log_{10}(\sqrt{|4\alpha_47_4 - \beta_4^2|} - \beta_4 - 27_4V_4) \end{bmatrix}$$
(180)

$$\frac{\alpha_4}{\gamma_4} - \frac{\beta_4^2}{2\gamma_4^2} < 0 . (178)$$

If the speed steadily decreases to V = 0 under the application of the brakes, the train will stop whom

$$t = t_4 + \frac{1}{28.6\sqrt{4\alpha_4^{7/4}-\beta_4^2}} \left\{ \sin^{-1} \left[\frac{2\gamma_4 V_4 + \beta_4}{\sqrt{4\gamma_4(\gamma_4 V_4^2 + \beta_4 V_4 + \alpha_4)}} \right] - \sin^{-1} \left[\frac{\beta_4}{\sqrt{4\alpha_4 \gamma_4}} \right] \right\} . \quad (183)$$

$$\frac{\alpha_5}{\beta_5} - \frac{\beta_5^2}{4\beta_5^2} < 0 \quad . \tag{14}$$

During braiding,

$$t = t_{+} + \frac{1}{20.6\sqrt{300/16-10.5}} \left\{ \sin \left[\frac{274\sqrt{4}+294}{\sqrt{474(24\sqrt{4}+24)}} \right] - \sin \left[\frac{274\sqrt{4}}{\sqrt{474(24\sqrt{4}+24)}} \right] \right\}$$
 (17)

provided
$$\frac{d_4}{d_4} - \frac{B_2^2}{d_{12}^2} > 0$$
; (17)

$$t = t_{+} + \frac{z}{z x V + \beta_{+}} - \frac{z}{z x V + \beta_{+}} \tag{1}$$

$$\frac{\alpha_b}{7_a} - \frac{\beta_b^2}{4\gamma_b^2} = 0 \quad ; \tag{17}$$

$$\frac{\partial_{a}}{\gamma_{a}} - \frac{\rho_{a}^{2}}{\gamma_{a}^{2}} < 0 \quad . \tag{9}$$

one to notestings ent volum 0 = V of measure the specification of the

busines, the busin will edop when

$$t = t_a + \frac{1}{28.6\sqrt{4\alpha_3t_4 - \beta_{a_a}^2}} \left\{ \sin^2 \left[\frac{2f_aV_a + \beta_{a_a}}{\sqrt{42t_a(f_aV_a^2 + \beta_{a_a}V_b + \alpha_a)}} \right] - \sin^2 \left[\frac{\beta_{a_a}}{\sqrt{44\alpha_3t_a}} \right] \right\}.$$

EXAMPLES

How much time will be consumed in the aboutless passed on

These examples are given for the purpose of illustrating the methods of applying the foregoing formulae. For simplicity, they are all computed for the same train.

Train Data

trantive affo

Number of cars, Motor cars, Trailers,	N = 6 3 3
Motors per motor car,	2
Number of motors,	M = 6
Weight of three motor cars,	3 X 28 = 84 tons
Weight of three trailers,	3 X 22 = 66 tons
Total weight of train,	T = 150 tons
Gross train weight per motor	150 % 6 = 25 tons
Area of projected cross-section	X = 110 sq.ft.
Average starting current per motor	350 amperes.
Motor Characteristics	Fig. 2

es the characteristic corres. Phys. 5. No. broaders effort.

= 8.8 seconds .

Ther formula (47) given

STATE LANGE

These examples are given for the purpose of illustrating the methods of amplying the foregoing formulae. For simplicity, they are all computed for the same train.

Train Data

warder of care Hotor cars, Trailers.

Motors per motor car,

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Weight of three motor cars.

weight of three trailers,

Total weight of trains

Gross train weight per motor 150 \$ 6 = 25 tons

Area of projected drose-section

Average starting current per motor 350 amperes.

Motor Oberectoristics

Z X 28 * 84 tons

anot 38 * SE X E

amod GGI = T

. di.ps OII = X

S .nig

1 - Starting

Now much time will be consumed in the starting period on level tangent track, that is, to accelerate the train from standstill to 17.2 miles per hour, the speed, on the motors' normal speed curve, corresponding to the average starting current of 350 amperes?

From the characteristic curves, Fig. 2, the tractive effort, corresponding to 350 amperes, is 5000 lbs. per motor. Hence, the tractive effort per tom of gross train weight is

Since the track is level and straight, and the brakes are not applied during starting. B = C = G = 0.

Substituting in equations (24), (25) and (26),

$$\alpha_{i} = 0.01(200 - \frac{50}{\sqrt{150}})$$

$$= 1.96 ,$$

$$\beta_{i} = 0.0003 0.3 \times 10^{-3} ,$$

$$\gamma_{i} = \frac{0.00002}{150} 110(1 + \frac{6-1}{10})$$

$$= 22 \times 10^{-6} .$$

Then formula (47) gives

$$t_s = \frac{2.30}{\sqrt{4 \times 1.96 \times 22 \times 10^{-6} + 0.09 \times 10^{-6}}} \begin{bmatrix} \log_{10}(2 \times 1.96 - 0.0003 \times 17.2 + 17.2 \sqrt{4 \times 1.96 \times 22 \times 10^{-6} + 0.09 \times 10^{-6}}) \\ -\log_{10}(2 \times 1.96 - 0.0003 \times 17.2 - 17.2 \sqrt{4 \times 1.96 \times 22 \times 10^{-6} + 0.09 \times 10^{-6}}) \end{bmatrix}$$

Substituting these values of hi , hi , hy and he in equations

= 8.8 seconds .

hy = - 760 +

naidwede - I

How made time will be command in the eterting period on level tempest track, that is, to accordante the train from standatill to 17.2 miles per hour, the speed, on the motors' normal speed curve, corresponding to the average sharting current of 350 ampers ?

From the description our rose, Fig. 2, the tractive effort, corresponding to 350 angures, is 5000 lbs. per motor. Tempe, the tractive effort per ton of gross brain weight in

F = 5000/25 = 200 lns. per ton.

whose the track is level and whroleht, and the brakes ore not explied during sparting. B=C=0=0

(35) and (35), (35) anothenge at galacticates

$$\alpha_{i} = 0.01(200 - \frac{50}{\sqrt{150}})$$

$$= 1.36 ,$$

$$\beta_{i} = 0.0003 0.3 \times 10^{3} ,$$

$$\gamma_{i} = \frac{0.00002}{150} 110(1 + \frac{6-1}{10})$$

$$= 22 \times 10^{6} .$$

the formula (47) gives

$$t_{6} = \frac{2.30}{\sqrt{4 \times 136 \times 22 \times 10^{-6} + 0.09 \times 10^{-6}}} \begin{bmatrix} \log_{6}(z_{x136} - 0.0003 \times 17.2 + 17.2 \sqrt{4 \times 136 \times 22 \times 10^{-6} + 0.09 \times 10^{-6}}) \\ -\log_{6}(z_{x136} - 0.0003 \times 17.2 - 17.2 \sqrt{4 \times 136 \times 22 \times 10^{-6} + 0.09 \times 10^{-6}}) \end{bmatrix}$$

= 8.8 seconds .

2 - Balancing Speed

What will be the balancing speed on a one per cent up-grade ? On the straight line AA, which approximates the motor power output current curve, Fig. 2,

P' = 49.4 when I = 100. and

Substituting these values in equation (2) gives

$$160 = h_1' + 325 h_2'$$

$$49.4 - h_1' + 100 h_2'$$

and

Solving these as simultaneous equations,

and

On the straight line BB, which approximates the tractive effort-current curve, Fig. 2, 0.03 /5.0 (20.0)

and

Substituting these values in equation (10) gives

and

Then

and

Substituting these values of hi, h, h, and h, in equations (20) and (21) gives

2 - Balanding Speed

hise

firms

bus

men

has

What will be the balancing aped on a one per cent up-grade ?

output current curve, Tig. 2,

entrangements of P = 160 when I = 325 - ...

end P' = 49.4 when I = 100.

Substituting these raives in equation (2) gives

180 = n + 385 n₂

49.6 = h1 + 100 h2 .

Solving these as simultaneous equations,

110.6 = 225 h

n's 0.4915

. 088.0 = te

On the straight line BE, which approximates the tractive

effort-current curve, Fig. 2,

2' = 4980 when I = 250

F' = 880 when I = 100 .

Substituting these values in squation (10) gives

4980 = hg + 350 hg

888 = h5 + 100 h4 .

. Troo = 390 pt

25.0 m = 16.4

. 057 - = MI

Substituting these values of his , hi in sealer enert guidatitude

sevia (IS) has (OS)

 $m = \frac{369800}{2} + \sqrt{\frac{(369800)^2}{4} + \frac{(10740)^3}{87}}$ Hemos, by forestine (78). = 15.0

Thus equation (22) gives _ 1.364 + 74.17 - 46.11

2 - Accelerating

and (60);

For one per cent grade.

Equations (50), (52), (53) and (54) give ported on a one per cont

$$\delta_{2} = 0.01 \left[\frac{-0.002}{150} 110 \left(1 + \frac{6-1}{10} \right) \right]$$

$$= -22 \times 10^{6} ,$$

$$\alpha_{2} = -\frac{0.01}{22 \times 10^{6}} \left[463.4 + 15.0 \left(20.0 + \frac{50}{\sqrt{150}} \right) \right]$$

$$= -374.8 \times 10^{3} ,$$

$$\beta_{2} = -\frac{10^{4}}{22} \left[0.03 \times 15.0 - \left(20.0 + \frac{50}{\sqrt{150}} \right) \right]$$

$$= 10.74 \times 10^{3} ,$$

$$\gamma_{4} = -\frac{10^{4}}{22} \left[\frac{0.002 \times 110 \times 15.0}{150} \left(1 + \frac{6-1}{10} \right) - 0.03 \right]$$

$$= -1.364 .$$

 $P_2 = \frac{1.364}{3} + 74.17(-1+1.5) - 46.11(-1-1.5)$

ting from stand-

Then equations (61) and (62) give

$$= \frac{10740 - \frac{(1.364)^2}{3}}{3} = \frac{10.74 \times 10^3}{3}$$

$$= -374800 + \frac{10740 \times 1.364}{3} - \frac{2(1.364)^3}{27}$$

$$= -369.9 \times 10^3$$

and

Relations (74) and (75) give

-13.58 +124.30 -15.0 m2 = 3(-19.58+124.30)2+2(-1364)(-13.58+124.30)+10740

Thus equation (22) gives

For one per cent grade.

(20.0 The late 0003 + 5

Squasions (50), (62), (65) and (64) give

$$\delta_2 = 0.01 \left[\frac{-0.002}{150} 110 \left(1 + \frac{6-1}{10} \right) \right]$$

$$\alpha_{2} = -22 \times 10^{6} ,$$

$$\alpha_{2} = -\frac{0.01}{22 \times 10^{6}} \left[463.4 + 15.0(20.0 + \frac{50}{\sqrt{150}}) \right]$$

$$\beta_{3} = -374.8 \times 10^{3} ,$$

$$\beta_{3} = -\frac{10^{4}}{22} \left[0.03 \times 15.0 - \left(20.0 + \frac{50}{\sqrt{150}} \right) \right]$$

$$I_{4} = \frac{10.74 \times 10^{3}}{22} \left[\frac{0.002 \times 110 \times 15.0}{150} \left(1 + \frac{\varepsilon - 1}{10} \right) - 0.03 \right]$$

Them beyond tone (13) and (62) give

- 269.9 X 10³.

melasions (74) and (76) give

$$m = \left[\frac{369900}{2} + \sqrt{\frac{(369900)^2}{4} + \frac{(10740)^3}{27}} \right]^{\frac{1}{3}}$$

$$= 74.17,$$

$$n = \left[\frac{369900}{2} - \sqrt{\frac{(369900)^2}{4} + \frac{(10740)^3}{27}} \right]^{\frac{1}{3}}$$

$$= -46.11.$$

Hence, by formulae (78), the halancing speed is

$$P_{0} = \frac{1.364}{3} + 74.17 - 46.11$$

$$= 28.52 \text{ miles per hour.}$$

3 - Accelerating

If the train is accelerated from the close of the starting period on a one per cent grade, how long after starting from stand-still will the speed become 28.0 miles per hour?

From the preceding exemples and equations (76), (78), (79) and (80),

$$t_{1} = 8.8 ,$$

$$V_{1} = 17.2 ,$$

$$V = 28.0 ,$$

$$\delta_{2} = -22 \times 10^{6} ,$$

$$\rho_{1} = 28.52 ,$$

$$\rho_{2} = \frac{1.364}{3} + 74.17(-\frac{1}{2} + j\frac{\sqrt{3}}{2}) - 46.11(-\frac{1}{2} - j\frac{\sqrt{3}}{2})$$

$$= -13.58 + j \cdot 24.30 ,$$

$$\rho_{3} = \frac{1.364}{3} + 74.17(-\frac{1}{2} - j\frac{\sqrt{3}}{2}) - 46.11(-\frac{1}{2} + j\frac{\sqrt{3}}{2})$$

$$= -13.58 - j \cdot 24.30 .$$

From equations (96), (97) and (98),

the last time if I is a a new by

$$m_2 = \frac{-13.58 + j24.30 - 15.0}{3(-13.58 + j24.30)^2 + 2(-13.58 + j24.30) + 10740}$$
$$= -0.00338 + j0.00182,$$

$$m = \begin{bmatrix} 369900 \\ 2 \\ 369900 \\ 369900 \\ 7 \end{bmatrix} + \sqrt{\frac{(369900)^2}{4} + \frac{(10740)^3}{27}} \end{bmatrix}^{1/3}$$

$$m = \begin{bmatrix} 369900 \\ 2 \\ 369900 \\ 2 \end{bmatrix} + \sqrt{\frac{(369900)^2}{4} + \frac{(10740)^3}{27}} \end{bmatrix}^{1/3}$$

$$m = \begin{bmatrix} 369900 \\ 2 \\ 369900 \\ 2 \end{bmatrix} + \sqrt{\frac{(369900)^2}{4} + \frac{(10740)^3}{27}} \end{bmatrix}^{1/3}$$

Hence, by formiles (78), the balencing speed is

$$Q_1 = \frac{1.364}{3} + 74.17 - 46.11$$
= 28.52 miles per hour.

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If the train is accordanced from the close of the starting ported on a one per sent grade, how long after start from stand-still will the speed books 28.0 miles per hour ?

Webs. the preceding executes and equations (VE), (VE), (VE),

.(00) han

$$t_{1} = 8.8 ,$$

$$V_{1} = 17.2 ,$$

$$V_{2} = 28.0 ,$$

$$\delta_{2} = -22 \times 10^{5} ,$$

$$\rho_{1} = 28.52 ,$$

$$\rho_{2} = \frac{1.364}{3} + 74.17(\frac{1}{2} + \frac{1}{3}) - 46.11(-\frac{1}{2} - \frac{1}{3})$$

$$= -13.58 + \frac{1}{2}4.30 ,$$

$$\rho_{3} = \frac{1.364}{3} + 74.17(-\frac{1}{2} - \frac{1}{3}) - 46.11(-\frac{1}{2} + \frac{1}{3})$$

$$\rho_{3} = \frac{1.364}{3} + 74.17(-\frac{1}{2} - \frac{1}{3}) - 46.11(-\frac{1}{2} + \frac{1}{3})$$

$$= -13.58 - \frac{1}{3}24.30 .$$

Firm agamalous (96), (69) and (90).

$$m_2 = \frac{-13.58 + j z 4.30 - 15.0}{3(-13.58 + j z 4.30)^2 + 2(-1358 + j 24.30) + 10740}$$

$$= -0.00338 + j 0.00182,$$

$$l_{2} = \frac{28.52 - 15.0}{3(28.52)^{2} + 2(-1.364)(28.52) + 10740}$$

$$= 0.00103 ,$$

$$n_{2} = \frac{-13.58 - j24.30 - 15.0}{3(-13.58 - j24.30)^{2} + 2(-1.364)(-13.58 - j24.30) + 10740}$$

$$= -0.00338 - j0.00182 .$$

Substituting these values in equation (104)

$$t = 8.8 + \frac{1}{-22 \times 10^{-6}} \left[0.00103 \log_{e} \frac{(28.00 - 28.52)}{(17.2 - 28.52)} + (-0.00338 + j0.00182) \log_{e} \frac{(28.00 + 13.58 - j24.30)}{(17.2 + 13.58 - j24.30)} + (-0.00338 - j0.00182) \log_{e} \frac{(28.00 + 13.58 + j24.30)}{(17.2 + 13.58 + j24.30)} \right]$$

$$t = 8.8 - \frac{10^6}{22} \left[0.00103 \log_e(\frac{0.52}{11.32}) + (-0.00338 + j0.00182) \log_e(41.58 - j24.30) - (-0.00338 + j0.00182) \log_e(30.78 - j24.30) + (-0.00338 - j0.00182) \log_e(41.58 + j24.30) - (-0.00338 - j0.00182) \log_e(30.78 + j24.30) \right]$$

For logarithms of the complex quantities, in general $log_e(x+jy) = log_e |x+jy| + jtan' \frac{y}{X} + 2jm\pi, \qquad m = integer,$ $= log_e(+\sqrt{x^2+y^2}) + jtan' \frac{y}{X} + 2jm\pi, \qquad j = +\sqrt{-1},$ $= 2.30 log_{io}(\sqrt{x^2+y^2}) + jtan' \frac{y}{X} + 2jm\pi.$

However, since m is any integer, this formula gives an infinite number of values of $log_c(x+jy)$. But only the principal values of definite integrals are concerned in speed-time determinations, so the last term, 2 j m π , may be neglected; that is let m=0. The resulting solution them is

t = 8.8 + 250.7 = 259.5 seconds .

approximately 4.00 minutes .

239,5 470.6

18 July

$$I_{2} = \frac{28.52 - 15.0}{3(28.52)^{2} + 2(-1.364)(28.52) + 10740}$$

$$= 0.00103 ,$$

$$n_{4} = \frac{-13.58 - j24.30^{2} + 2(-1.358 - j24.30) + 10740}{3(-13.58 - j24.30)^{2} + 2(-1.358 - j24.30) + 10740}$$

$$= -0.00338 - j0.00192 ,$$

(501) nolferop at america const prituation (504)

$$t = 8.8 + \frac{1}{-22 \times 10^{3}}$$

$$+(-0.00103 \log_{e} \frac{(28.00 - 28.52)}{(17.2 - 28.52)}$$

$$+(-0.00338 + j 0.00182) \log_{e} \frac{(28.00 + 13.58 - j 24.30)}{(17.2 + 13.58 - j 24.30)}$$

$$+(-0.00338 - j 0.00182) \log_{e} \frac{(28.00 + 13.58 + j 24.30)}{(17.2 + 13.58 + j 24.30)}$$

$$t = 8.8 - \frac{10^6}{22} \left[0.00103 \log_e(\frac{0.52}{11.32}) + (-0.00338 + j0.00182) \log_e(4138 - j24.30) + (-0.00338 + j0.00182) \log_e(4138 - j24.30) + (-0.00338 - j0.00182) \log_e(41.58 + j24.30) + (-0.00338 - j0.00182) \log_e(30.78 + j24.30) + (-0.0038 - j0.00182) + (-0.003$$

For logarithms of the complex curvities, in consent $log_{c}(x+jy) = log_{c}(x+jy) + jtan'\frac{y}{X} + 2jm\pi, \quad m = integer,$ $= log_{c}(+\sqrt{x^{2}+y^{2}}) + jtan'\frac{y}{X} + 2jm\pi, \quad j = +\sqrt{-1}.$ $= 2.30 log_{c}(\sqrt{x^{2}+y^{2}}) + jtan'\frac{y}{X} + 2jm\pi.$

However, aimag at the easy interpret, this forests offers an infinite number of values of $log_{\epsilon}(K+jy)$. This only the principle values of definite independent are concerned in appeal-time detections, so the tens tens, if may, may be negliarized; that is let m=0. The resulting solution than is

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approximately

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4 - Coasting

If the power is shut off when the speed reaches 26.0 miles per hour, and the train is allowed to coast up the one per cent grade, how long after the train started will the speed become 10.0 miles per hour?

Equations (116), (117) and (118) give

$$\alpha_{3} = 0.01 \left(20.0 + \frac{50}{\sqrt{150}}\right)$$

$$= 0.2408 ,$$

$$\beta_{3} = 0.0003 ,$$

$$\gamma_{3} = \frac{0.00002}{150} 110 \left(1 + \frac{6-1}{10}\right)$$

$$= 22 \times 10^{-6} ,$$

$$\frac{\alpha_{3}}{\gamma_{3}} - \frac{\beta_{3}^{2}}{4\gamma_{3}^{2}} = \frac{0.2408}{22 \times 10^{-6}} - \frac{0.09 \times 10^{-6}}{4 \times 484 \times 10^{-6}}$$

$$= 10.9 \times 10^{3} > 0 .$$

= 102×103 > 0.

Therefore, formula (135) applies and

$$t = 239.5 + \frac{1}{28.6\sqrt{4\times0.2408\times22\times10^{-6}-0.09\times10^{-6}}} \begin{bmatrix} \sin^{-1} \frac{2\times22\times10^{-6}\times28.0+0.0003}{\sqrt{4\times22\times10^{-6}(22\times10^{-6}\times28.0+0.0003\times28.0+0.2408)}} \\ -\sin^{-1} \frac{2\times22\times10^{-6}\times28.0+0.0003}{\sqrt{4\times22\times10^{-6}(22\times10^{-6}\times10+0.0003\times10.0+0.2408)}} \\ = 239.5 + 70.6 .$$

= 5.17 minutes .

= 310 seconds ,

= 5.25 minutes.

A - Counting

If the power is east off when the speed reselve 23.0 miles per hour, and the train is allowed to open up the one per cout grade. how long ofter the train started will the speed become 15.0 miles par hour?

mendions (110), (117) and (118) give

$$\alpha_{3} = 0.0I(20.0 + \frac{50}{\sqrt{150}})$$

$$= 0.2408 ,$$

$$\alpha_{5} = \frac{0.00002}{150}IIO(I + \frac{6-1}{10})$$

$$= 22 \times IO^{6} ,$$

$$\alpha_{5} = \frac{62}{22 \times IO^{6}} ,$$

$$\alpha_{5} = \frac{62}{22 \times IO^{6}} = \frac{0.09 \times IO^{6}}{4 \times 484 \times IO^{6}}$$

$$= 10.9 \times IO^{3} > 0 .$$

Thorotope (188) altered and one

$$t = 239.5 + \frac{2 \times 22 \times 10^{10} \times 26.0 + 0.0003}{26.5 \times 10^{10} \times 26.0 \times 10^{10} \times 26.0 \times 10^{10} \times 26.0 \times 10^{10} \times 26.0 \times 10^{10} \times 1$$

- = 239.5 +70.6
- = 310 seconds ,
- = 5.17 minutes .

5 - Braking

From the time the speed reaches 10.0 miles per hour until
the train stops, an average braking effort of 200 pounds per tom of
gross train weight is applied by friction brakes, the grade continuing
at +1.00 per cent. How much time will have been consumed in making
the run from start to stop?

From equations (165), (166) and (167),

$$\alpha_{4} = 0.01(200 + 20.0 + \frac{50}{\sqrt{150}})$$

$$= 2.2408,$$

$$\beta_{4} = 0.0003,$$

$$\gamma_{4} = \frac{0.00002}{150}110(1 + \frac{6-1}{10})$$

$$= 22 \times 10^{-6},$$

$$\frac{\alpha_{4}}{1_{4}} - \frac{\beta_{4}^{2}}{4\gamma_{4}^{2}} = \frac{2.2408}{22 \times 10^{-6}} - \frac{0.09 \times 10^{-6}}{4 \times 484 \times 10^{-2}}$$

$$= 102 \times 10^{3} > 0.$$

Therefore formula (183) applies and

$$t = 310 + \frac{1}{z_{8.6}\sqrt{4\times z.2408\times z.2\times16^6 - 0.09\times16^6}} \left[si\vec{n}' \frac{2\times 22\times16^6\times10.0 + 0.0003}{\sqrt{4\times z.2\times16^6(22\times16^6\times16.0^6\times16.0^6\times10.0 + 2.22408)}} - si\vec{n}' \frac{0.0003}{\sqrt{4\times z.2408\times z.2\times16^6}} \right]$$

= 310 + 4.8

= 315 seconds

= 5.25 minutes.

From the time to appeal received like miles you have muchi

the train stops, an everage bracing effort of 200 youses per ten of gross train weight is applied by Erletion brokens, the grade constants at #2.00 per cent. Now much time will have been communed in meking the run from where to whop ?

From squations (165), (166) and (167),

$$\alpha_{4} = 0.01(200 + 20.0 + \frac{s_{0}}{\sqrt{150}})$$

$$= 2.2408,$$

$$\beta_{4} = 0.00003,$$

$$\gamma_{4} = \frac{0.00002}{150}110(1 + \frac{6-1}{10})$$

$$= 22 \times 10^{6},$$

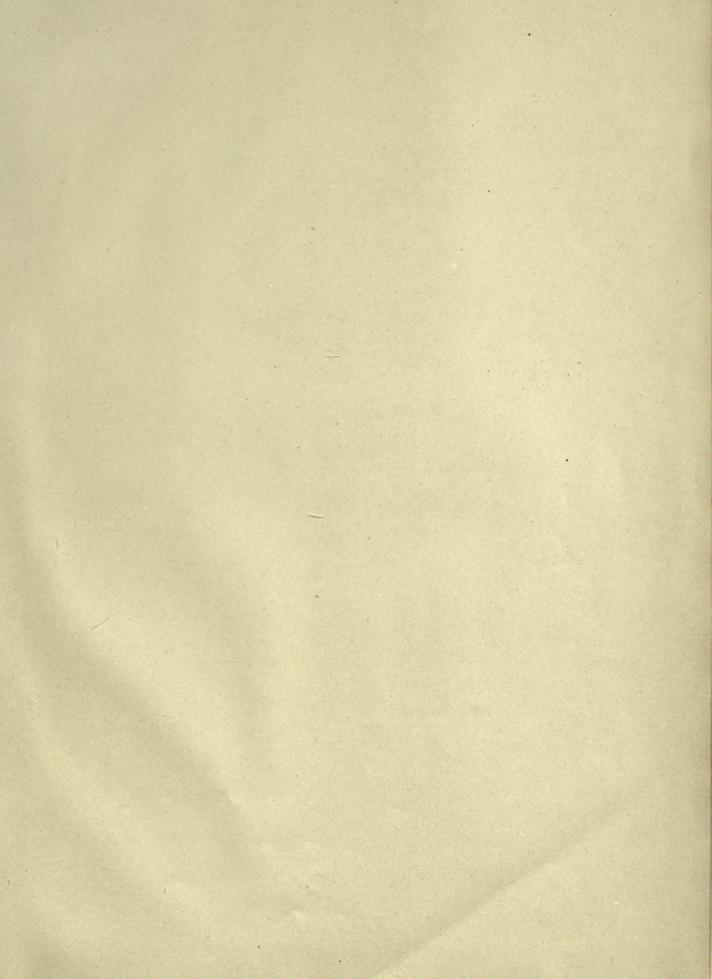
$$\frac{\alpha_{4}}{\gamma_{4}} = \frac{\beta_{4}^{2}}{22 \times 10^{6}} = \frac{0.09 \times 10^{6}}{4 \times 464 \times 10^{6}}$$

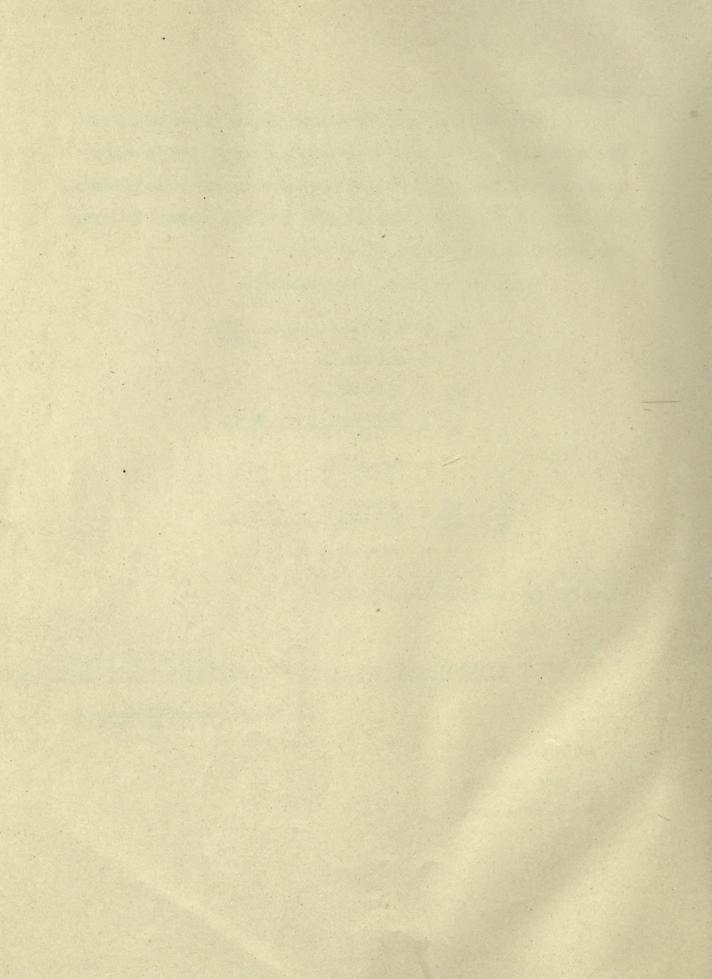
$$= 102 \times 10^{3} > 0,$$

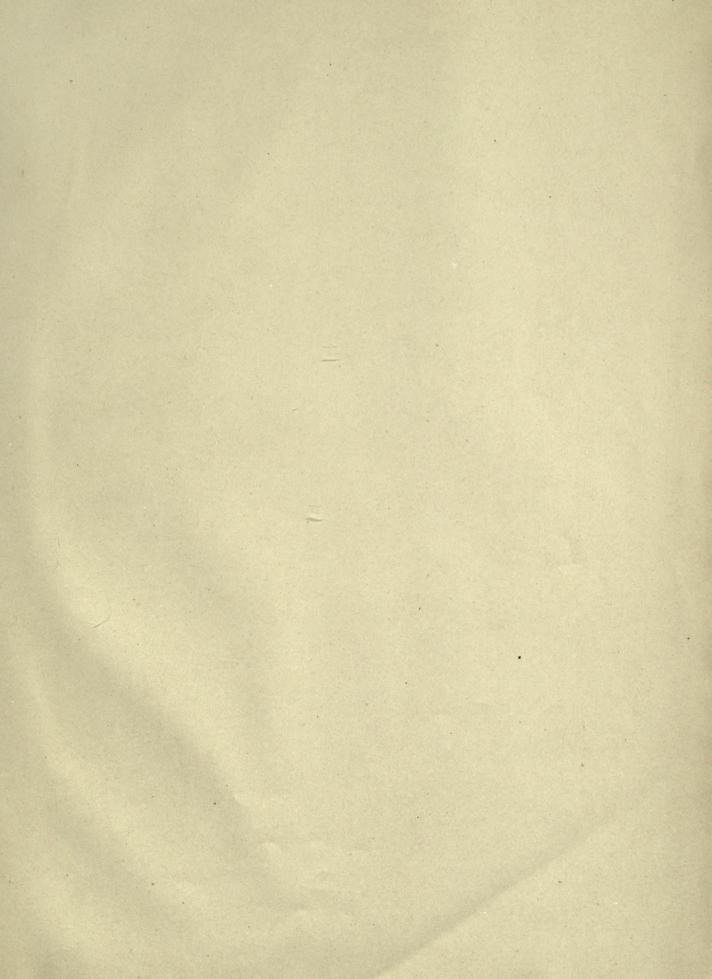
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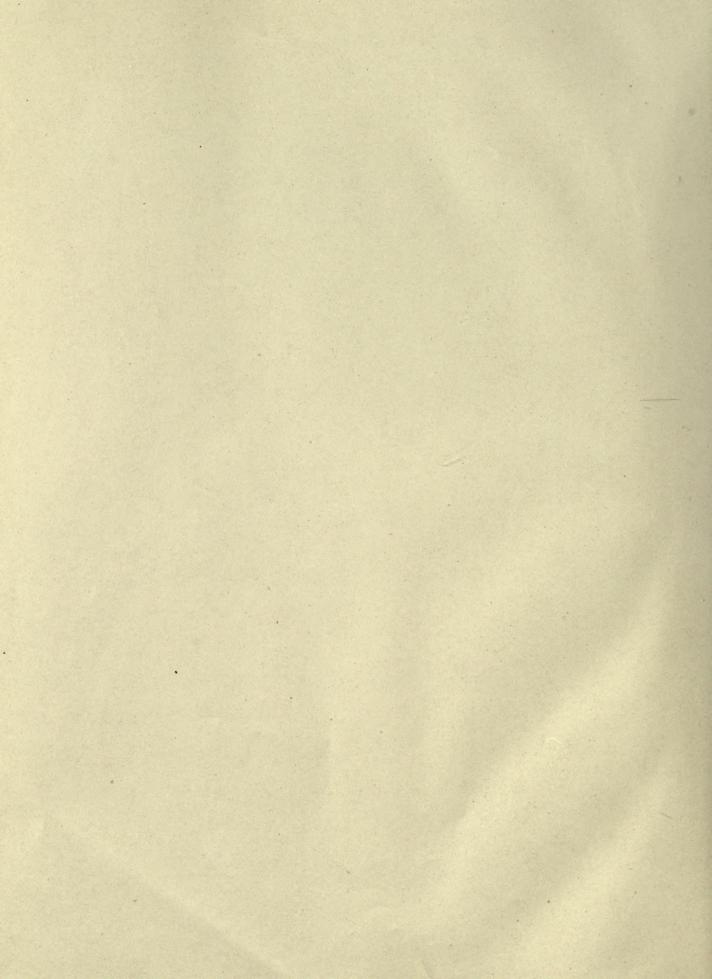
$$t = 310 + \frac{1}{z_{8.6}\sqrt{4xz.2408x22x10^{5} - 0.09x10^{5}}} = sin^{4} \frac{2x22x10^{5}x10.0 + 0.0003}{\sqrt{4x22x10^{5}(22x10^{5}x10.0 + 0.0003x0.0 + 2.2408}} - sin^{4} \frac{0.0003}{\sqrt{4x2.2408x22x10^{5}}}$$

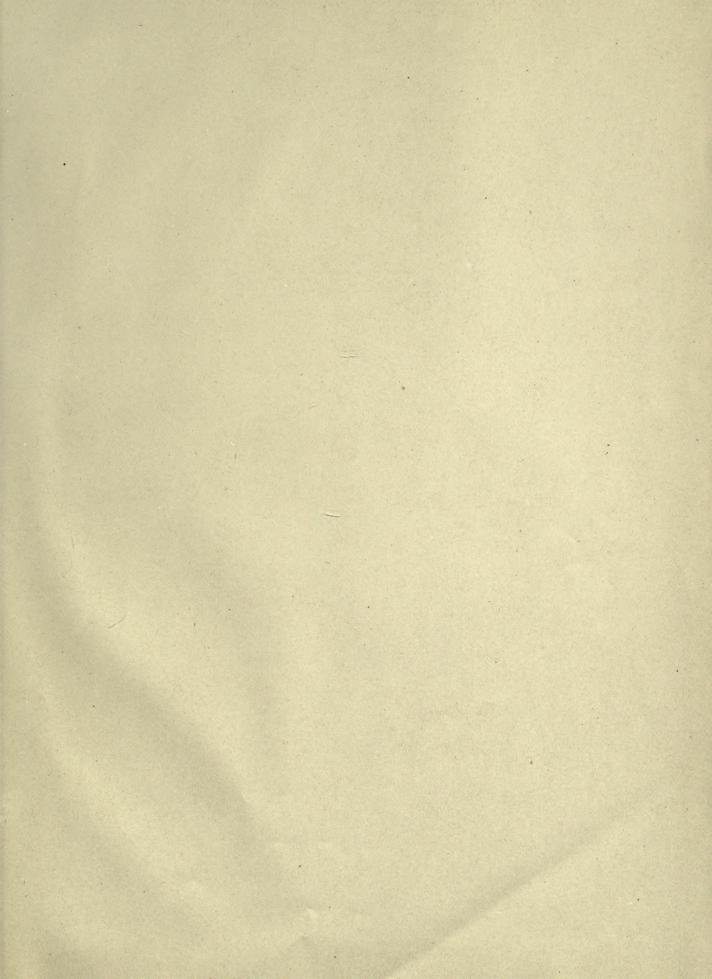
- = 310 + 4.8
- # 315 seconds
- = 5.25 minutes.

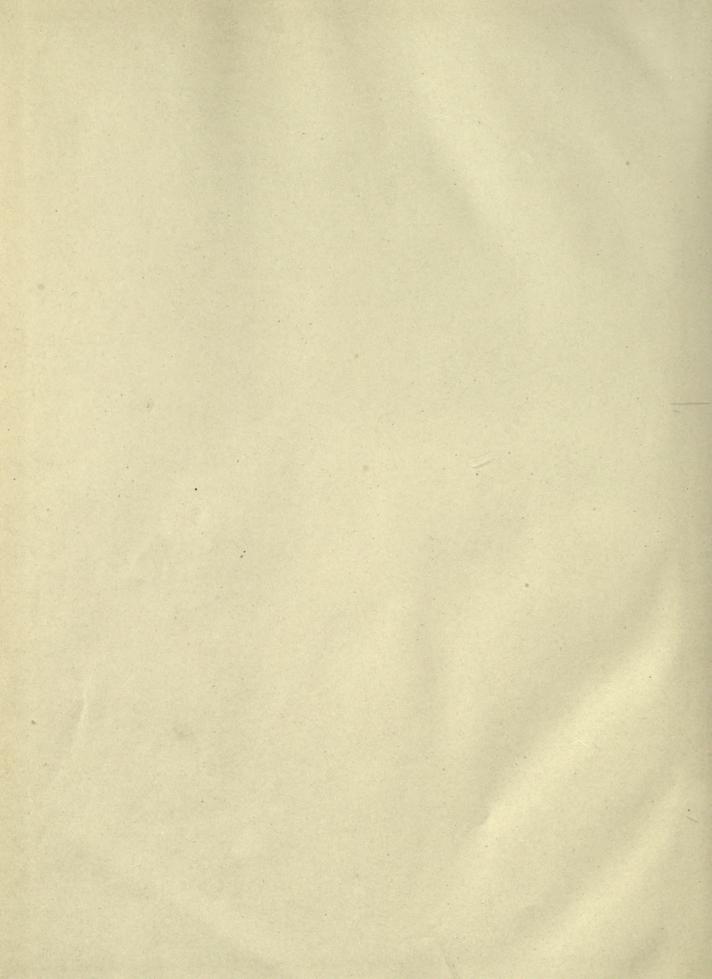


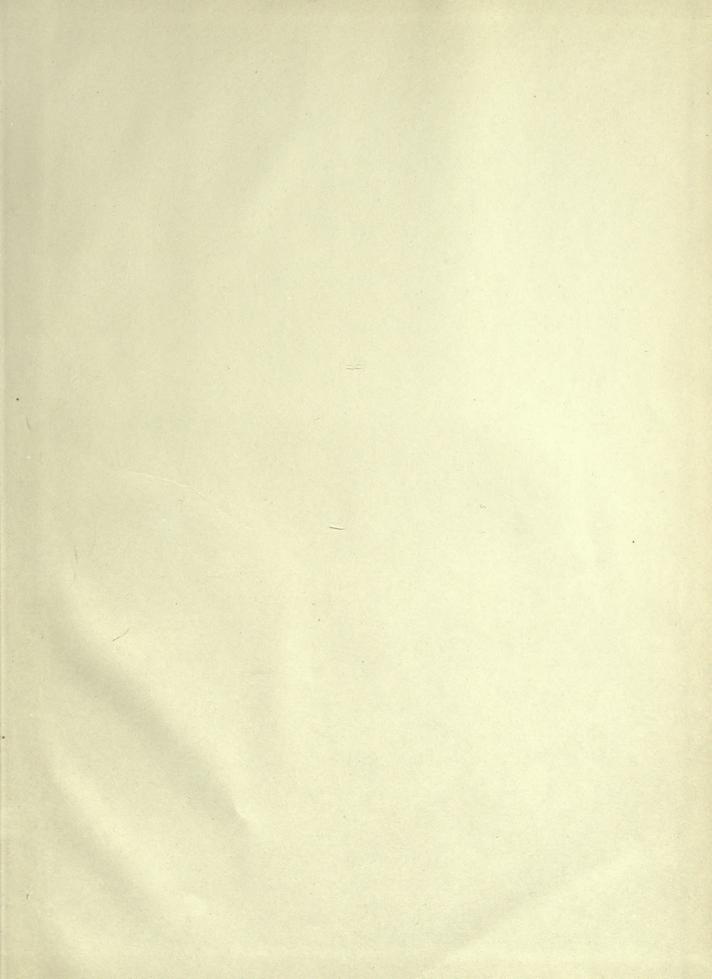












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